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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

Volume XL Number 7

Edited by William David Reeve

Guidance Report of the Commission on Post-War Plans*

I. To the High School Student

Why should I study mathematics? What good will mathematics be to me? Perhaps you have asked yourself these two questions. If so, you have a right to good answers, and you will find them in the following pages.

The purpose of this report is to describe for you some occupations in which mathematics is important. You will learn about the amount and kind of mathematics that is used and the school level at which it should be taken.

Many people discover only after they have begun to work that their job requires a type of training in mathematics they have not had. At times they are able to get this by attending night school, taking correspondence courses, or studying at home. However, this way is costly and time consuming. Also, it is very risky, for they may not have learned some things well enough.

There is no reason why you should make an error of this type. If the occupation in which you are interested is not mentioned in this pamphlet, ask your teacher for the necessary information. You can be quite sure that mathematics in some form is needed.

It isn't likely that you will want to read all of this report. However, you may find some parts that will interest you very much especially if you discuss them not only with your teachers, but with your parents and friends. Then you are almost certain to get a good answer for your question, "Why should I study mathematics?"

II. Mathematics for Personal Use

Fred looked up from his trigonometry when he heard the drone of a great sky liner. Far up in the blue sky, he saw it merge into the red and orange of the setting sun. Soon it would be dark. But ever onward, riding the beam, roars the great liner while the navigator, with his computer (a modified slide rule), his charts, his radar, his radio aids, his sextant, and his mathematical tables, stands by alert and ready for any emergency. The fifty or more passengers go to sleep with a feeling of security. They ride with mathematics!

Fred watched until the great liner vanished in the golden clouds. "There," he said, "goes my algebra, geometry and trigonometry; for without them there would be no airplanes or navigation."

Navigation is only one of a great many fields and professions for which mathe-

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matics is basic—engineering, medicine, research, chemistry, physics and the like. In fact, it would be easier to name twenty professions for which a strong mathematical background is desirable than to think of two in which such preparation would not be useful. If you take the long road to a scientific career it means years of systematic study of mathematics; your study of algebra, geometry and trigonometry is just a beginning.

By this time you may be thinking, "I am not going to be an engineer or a doctor. I'm not the least bit interested in science; in fact, I'm not even thinking about college. So I'll skip mathematics."

Wait a minute! You are living in a machine world. You want to read the newspapers intelligently. You talk to people about the adventures of science. You listen critically to the radio with an ear for the "whole truth and nothing but the truth." Without mathematics you will not know the score in a scientific age. Even if you are not headed for science, you nevertheless have a *personal* use for mathematics.

At this point a word of caution is in order. In great grandfather's day, a person could get along fairly well if he didn't know how to read. And he got along somehow if he could do only a little figuring with whole numbers and common fractions, and knew a bit about decimals and per cents. But that won't do today! This shrinking world doesn't mean too much to a person who can't read. In like manner a great deal more mathematics is required for personal use today than was needed in pioneer times.

Let's take a look at the mathematics that you will use in the home, in community life, in reading newspapers and magazines, and in everyday occupations.

Mathematics in the Home—All through your life you will be figuring money and money values. There's no denying—you'll need a good background in arithmetic for the bookkeeping job of managing some kind of a household budget. As a consumer you must be able to figure rapidly the "best buy" from goods that look pretty much alike.

Installment buying is a booby trap for many persons. For example: Suppose you try this problem on ten or more grown-ups: *A second-hand bicycle is marked $20 cash. Sally buys this bicycle by paying $2 down and $2 per week for 10 weeks. What is the rate of interest per year that Sally pays?* You probably say "10%." So will some grown ups. A few may say 50%. A good answer is 94.5%, another is *104% per year.** When you avoid pitfalls by figuring interest rates on installment buying accurately, a good solid footing in mathematics comes in mighty handy. It's really surprising how much money can be saved through wise buying.

In the home, the man-of-the-house has to deal with figures when he borrows money, checks his bank account, finances a home, pays interest on the mortgage, makes an investment, or pays the taxes.

Likewise the housewife uses figures when she manages the food budget, pays bills promptly to get a discount, makes Jane a dress, weighs the baby, checks calories in the daily menu, changes a recipe, or plans a party.

The long and short of it is that an efficient home uses mathematics in many ways. But the members of the family are probably not much more aware of it than they are of the air they breathe.

Mathematics for Citizenship—As a citizen in a community your first job is to use your mathematics in providing for the security of your home and family. In what way? Through wise buying of various forms of insurance, as for example, life, health, accident, unemployment and automobile. Prepare for what the future may bring. Remember that the time may come when you can't earn your living; so keep an eye on security in old age.

With sufficient mathematical back-

* There are several acceptable methods of computing interest per year in problems like this one.

ground you can understand local, state, and federal taxation and something about the collection and use of public funds.

You will read a great deal of statistics dealing with public welfare, national defense, and government loans. And then there are community chests, churches, schools, and other agencies that affect community living. As a member of some group dealing with such problems you will need not only *common* sense, but also *number* sense. Without it you may not be able to do your fair share in community projects.

For Intelligent Reading of Newspapers, Magazines, and Bulletins—It may be that you will not wear out many pencils in computation. But you will probably read a vast number of pages that you can never quite follow unless you understand the meaning of mathematical terms that are used again and again. This mathematical language includes such words as unit, ratio, average, percentage, proportion, variable, constant, dimension, and formula.

And there is something else. You should be able to read simple tables, line graphs, bar graphs, circle graphs, and pictograms. Books, magazines, and bulletins are using these devices more and more to make the ideas clear to the reader.

The daily newspaper brings to our homes figures and statistics on employment, overtime schedules, opinion polls, speed limits, indexes, rainfall, bank rates, freight rates, stock and bond summaries, national debts, taxes, death rates, speed limits, bowling averages, baseball standings, weather reports, and various calculations of a machine age. As someone has said "ratios, limits, and accelerations are . . . photographed upon every page of our existence . . . the man of the machine age is a calculating animal."

For Everyday Occupations—Ordinary jobs require some mathematical knowledge and skill. When you enter your first job your employer will assume that you have a fair mathematical background. You will be handicapped if you can't keep a simple account, make change, or solve common problems of communications, travel, and transportation.

Business requires that goods be kept moving and not grow stale in stock. This means that mathematical inventories must be made to guide further action. The office force is responsible for money, stock, and credit; it also keeps the books and analyzes the business.

Craftsmen must use their measuring instruments accurately and solve their problems in a hurry. Most of them learn decimals on the job and wish they had learned them in school.

Estimators must figure close enough to get the job when competing with others. At the same time they must figure high enough to make a profit for the firm for which they work.

Successful farmers must know enough mathematics to estimate net profits. How is this done? By figuring gross receipts and deducting necessary expenses which, might include: trucking, fertilizers, labor, seeds, sprays, feed, and what not. Farmers can't understand government bulletins written for them unless they understand how to read graphs and understand clearly the meaning of such mathematical ideas as parity and indices. We have been talking about successful farmers. There are still lots of farmers who don't keep books and they have been known to operate for years without realizing that they are broke.

Now for a pleasant surprise! All these computations that sound so difficult are usually based on simple arithmetic dealing with whole numbers, fractions, decimals, percentages, and common units of measure.

But—and here's a slight hitch—*successful* workers in everyday jobs read technical books and articles which include basic mathematics that goes beyond mere computation. To get rapid promotions on semi-technical jobs in industry you have to be something more than merely a good computer.

Computation with whole numbers, decimals, fractions, and per cent comes first but there is something more that you ought to know. How much mathematics is a "must" for every citizen? It's time to get that straightened out. Here is a Check List of 29 questions. If you can say "yes" to nearly all of them you can feel pretty secure when it comes to dealing with the problems of everyday affairs.

THE CHECK LIST

1. *Computation.* Can you add, subtract, multiply, and divide effectively with whole numbers, common fractions, and decimals?
2. *Percents.* Can you use percents understandingly and accurately?
3. *Ratio.* Do you have a clear understanding of ratio?
4. *Estimating.* Before you perform a computation, do you estimate the result for the purpose of checking your answer?
5. *Rounding numbers.* Do you know the meaning of significant figures? Can you round numbers properly?
6. *Tables.* Can you find correct values in tables; e.g., interest and income tax?
7. *Graphs.* Can you read ordinary graphs: bar, line and circle graphs? the graph of a formula?
8. *Statistics.* Do you know the main guides that one should follow in collecting and interpreting data; can you use averages (mean, median, mode); can you draw and interpret a graph?
9. *The nature of a measurement.* Do you know the meaning of a measurement, of a standard unit, or the largest permissible error, of tolerance, and of the statement that "a measurement is an approximation"?
10. *Use of measuring devices.* Can you use certain measuring devices, such as an ordinary ruler, other rulers (graduated to thirty-seconds, to tenths of an inch, and to millimeters), protractor, graph paper, tape, caliper micrometer, and thermometer?
11. *Square root.* Can you find the square root of a number by table, or by division?
12. *Angles.* Can you estimate, read, and construct an angle?
13. *Geometric concepts.* Do you have an understanding of point, line, angle, parallel lines, prependicular lines, triangle (right, scalene, isosceles and equilateral), parallelogram (including square and rectangle), trapezoid, circle, regular polygon, prism, cylinder, cone, and sphere?
14. *The 3-4-5-relation.* Can you use the Pythagorean relationship in a right triangle?
15. *Constructions.* Can you with ruler and compasses construct a circle, a square, and a rectangle, transfer a line segment and an angle, bisect a line segment and an angle, copy a triangle, divide a line segment into more than two equal parts, draw a tangent to a circle, and draw a geometric figure to scale?
16. *Drawings.* Can you read and interpret reasonably well, maps, floor plans, mechanical drawings, and blueprints? Can you find the distance between two points on a map?
17. *Vectors.* Do you understand the meaning of vector, and can you find the resultant of two forces?
18. *Metric system.* Do you know how to use the most important metric units (meter, centimeter, millimeter, kilometer, gram, kilogram)?
19. *Conversion.* In measuring length, area, volume, weight, time, temperature, angle, and speed, can you shift from one commonly used standard unit to another widely used standard unit; e.g., do you know the relation between yard and foot, inch and centimeter, etc ?
20. *Algebraic symbolism.* Can you use letters to represent numbers; i.e., do you understand the symbolism of algebra—do you know the meaning of exponent and coefficient?
21. *Formulas.* Do you know the meaning of a formula—can you, for example, write an arithmetic rule as a formula, and can you substitute given values in order to find the value for a required unknown?
22. *Signed numbers.* Do you understand signed numbers and can you use them?
23. *Using the axioms.* Do you understand what you are doing when you use the axioms to change the form of a formula or when you find the value of an unknown in a simple equation?
24. *Practical formulas.* Do you know from memory certain widely used formulas relating to areas, volumes, and interest, and to distance, rate, and time?
25. *Similar triangles and proportion.* Do you understand the meaning of similar triangles, and do you know how to use the fact that in similar triangles the ratios of corresponding sides are equal? Can you manage a proportion?
26. *Trigonometry.* Do you know the meaning of tangent, sine, cosine? Can you develop their meanings by means of scale drawings?
27. *First steps in business arithmetic.* Are you mathematically conditioned for satisfactory adjustment to a first job in business; e.g., have you a start in understanding the keeping of a simple account, making change, and the arithmetic that illustrates the most common problems of communications and every day affairs?
28. *Stretching the dollar.* Do you have a basis for dealing intelligently with the main problems of the consumer; e.g., the cost of borrowing money, insurance to secure adequate protection against the numerous hazards of life, the wise management of money, and buying

with a given income so as to get good values as regards both quantity and quality?

29. "*Proceeding from hypothesis to conclusion.* Can you analyze a statement in a newspaper and determine what is assumed, and whether the suggested conclusions really follow from the given facts or assumptions?

This little kit of tools that you have been playing with in the above Check List is enormously useful in solving some very complicated problems in present-day life. It is the basic mathematics needed in a great many jobs. You will admit that the list is short and we hope that some day it will seem absurdly simple! Nevertheless most businessmen who hire high school graduates would be greatly pleased if workers came to them "knowing this stuff fairly well."

And now we come to an important question. What mathematics should you study in order to get what you need for personal use? In some schools a few of the best students can get all this by the end of the eighth grade. Most pupils, however, would need to continue general mathematics for one or two years, in order to master what the Check List calls for.

If you are thinking about industry then you are lucky if you are in a school big enough to provide a year or more of shop mathematics.

You can learn some of the items in the check list by taking a course in business arithmetic. However, you can learn the same things and more in a newer course called "Consumer Mathematics." In recent years more and more schools have added this course as an elective in the eleventh and twelfth grades.

Anyone who takes four years of traditional mathematics (algebra, geometry, etc.) will get most of the basic mathematics needed for personal use. There would, no doubt, be some gaps as these courses are now organized and taught. At any rate if the mathematics for personal use is all that you want, traveling the traditional road would be a pretty expensive way of getting it. You would probably be wiser to take some of the more general courses we have described.

It is unfortunate that large numbers of students fail to realize the importance of gaining skills and understandings in mathematics *while they are in school.* Such students find out too late that they could have, and should have, mastered the fundamentals of mathematics that are required in most jobs. For them remain the evening schools and part-time classes or home study if they desire to learn the fundamentals they lack.

It wouldn't be fair to you if we ended this discussion of mathematics for personal use without mentioning *enjoyment* and *cultural* values. There is much more in the study of mathematics than the vocational value. Throughout the centuries there have been many persons who have enjoyed mathematics for its own sake. There is a precious cultural value that comes to those who learn to appreciate the contribution of mathematics to civilization—to the progress of man. You are entitled to know (1) that few students if any who are competent in mathematics have ever regretted time spent in learning the subject, (2) that mathematics is an easy subject if well taught and the structure is built carefully step by step, and (3) that there are few subjects which students like better than mathematics, provided it is well taught. So, if you enjoy mathematics, take no thought of the vocational uses in the tomorrow—sufficient for this day is the *good* thereof.

III. Mathematics Used by Trained Workers

In general the mathematics used by skilled and semi-skilled workers does not require training beyond the high school. If you have a good number sense and fine training in the mathematics courses in high school, you are qualified to hold anyone of hundreds of jobs that require applied arithmetic and the mathematics outlined in the Check List in the preceding

section. Brief descriptions of a few such opportunities are given below.

1. *Bookkeepers*

A bookkeeper is a person in a business office who keeps track of all business transactions. His is a pretty big job! He has to keep cash books, ledgers, expense accounts, invoices, and vouchers straight. Not to mention notes, payrolls, inventories, and bank statements which he has to check.

The bookkeeper's figuring is done with the "common garden variety of arithmetic"—whole numbers, fractions, decimals, and per cents. But he's got to be on his toes. He must be able to figure accurately, rapidly, and with confidence. And it's up to him to spot errors, take short cuts, estimate results, and check the work. Luckily, enough calculating machines, mathematical tables, and graphs are usually around to lend him a helping hand.

The business departments of public high schools and private business colleges give training for this kind of work. The mathematics that you would need in bookkeeping includes not only basic arithmetic, but also business arithmetic, bookkeeping, and some accounting. Additional training in English, economics, and business law helps a lot. A bookkeeper with more training may advance to an accountant. Just where bookkeeping leaves off and accounting begins is not easy to say.

Now for a bit of fatherly advice. Unless you have good general ability and are far better than average when dealing with figures, steer clear of bookkeeping. Many students with low ability take commercial arithmetic in high school with the futile hope that they will be all set for bookkeeping. It just won't happen!

2. *Clerical Workers*

Take a look at this table. You'll admit, won't you, that you didn't know the extent to which men are employed as secretaries, stenographers, typists. More than women! If you have ability, one step toward a job with a big fat salary might be to serve for a while as a secretary to someone in a large corporation.

CLERICAL WORKERS EMPLOYED
(1940 CENSUS)

	Men	*Women*
Agents, miscellaneous	92,000	9,000
Attendants, physicians' and dentists' offices	31,000	28,000
Attendants and assistants, library	20,000	7,000
Ticket, station and express agents	40,000	2,000
Collectors, bill and account	44,000	3,000
Office machine operators	64,000	51,000
Shipping and receiving clerks	230,000	9,000
Stenographers, typists, and secretaries	1,175,000	988,000
Clerical and kindred workers, miscellaneous	1,974,000	630,000

The table also shows the types of workers in the clerical field. By observing the total number of men and women employed you get some idea of the importance of this work. In recent years the need for clerical workers has been great. In some places recruiting agents have been making the rounds of schools to pick up students as soon as they graduate.

When business and industry boom, clerical workers are in demand. Even in hard times the secretary that is tops will likely have a good job.

Not every secretary marries the boss's son or daughter. For the long pull you had better rest your future as a clerical worker on fine personal qualities, mental alertness, and good training.

As far as mathematics is concerned, we say once more "Bear down on the items in the Check List pages 318-319." That's about all the mathematics you will need.

Oh! we almost forgot. Instruction and practice with computing, punch card, and card sorting machines will help. And by the way, it isn't correct to call a clerical worker who operates machines a "statistician." As you will see later, a statistician is something else.

If you are in a school that does not offer

business arithmetic, general mathematics, and practice with machines, perhaps you can pick these up some time in night school or by taking correspondence courses.

3. *Craftsmen*

How many men do you suppose earn their living at some skilled trade? The answer is more than six million! There are scores of skilled trades. It might be fun for you to see how many you can name—bricklayer, carpenter, dressmaker, electrician, plumber, mechanic etc.

Think twice before you pass up all of these jobs. If you have a flair for a skilled trade, you will get satisfaction all your life in the doing of the work. Pay has been increasing and in fact is better than that of white collar workers. The hours will likely drop still further. You would have time and energy to work in the garden or at some hobby. People appreciate and hold the skillful craftsman in high respect.

How do you get into one of these skilled trades? Apprenticeship is the road to most of them. In general an apprentice is a learner of not less than 16 years of age who works under the direct supervision of a skilled worker or journeyman.

If you were an apprentice, you'd get a wage that would be automatically increased at regular intervals. At the end of a certain number of years you would complete training and thus become a craftsman or journeyman.

How do you get to be an apprentice? That varies in the different trades and according to local situations. In some places you may need to have a relative in the trade. In many places you would need to apply to the head of the local union.

Sometimes mechanical aptitude tests are given for entrance to apprenticeship. This is done at the Government Printing Office, U. S. Navy Yard, and in certain large industries. Such tests include questions on elementary trigonometry, simple machines, interpretation of drawings and diagrams, and measurement of figures and solids.

There are scores of apprenticeable trades. Of these, the following require trade mathematics, especially measurement in some form.

APPRENTICEABLE TRADES THAT REQUIRE MATHEMATICS*

Baker (hand) 2
Blacksmith 4
Boilermaker 4
Bricklayer 3
Cabinetmaker 4
Carpenter 4
Cement finisher 2
Clothes-designer 4
Compositor 5–6
Cook 2
Cooper 3
Coppersmith 4
Coremaker 4
Dental mechanic 4
Diemaker 4
Draftsman 4
Dressmaker 2
Drop-forger 4
Electrician:
Aircraft 4
Automotive 4
Building 4–5
Linesman 4
Electrotyper 5
Engraver 7
Furrier 3
Garment cutter 3
Glazier 3
Granite polisher 2
Heat treater 4
Instrument maker 4
Jeweler 4
Lather 2
Leather worker 3
Lens grinder 4
Lithographer 4
Machinist 4
Meat cutter 3
Mechanic 4
Mechanic Miscl. 4
Millman 4
Millwright 4
Model maker 4
Mold loftsman 4
Molder 4
Offset-pressman 4
Operating engineer 2–3
Ornamental iron worker 4
Painter and paperhanger 3
Paper maker 4
Patternmaker, metal 5
Patternmaker, wood 5
Pipe-fitter 5
Plasterer 4
Plumber 5
Pottery-kilnman 3
Pressman 5
Printer 5
Radio technician 4
Sheetmetal worker 4
Shipfitter 4
Shipwright 4
Stationary engineer 4
Steam fitter 5
Stone carver 4
Stone mason 3
Structural steel w. 2
Tailor 4
Tile setter 3
Toolmaker 4
Upholsterer 4
Watchmaker 4
Wire weaver 4
Woodworker, aircraft 4

* Numbers after each trade indicate the necessary learning period in years.

As a skilled worker you must have a good knowledge of mathematics especially as applied to the particular trade in which you are employed. There measurement is a

first consideration. Each trade has special units of measure with appropriate measuring instruments. For example, in the machine trades, craftsmen use such instruments as the steel scale, micrometer, vernier calipers, inside and outside calipers, and gauge blocks.

The beginner in any trade must master certain fundamentals of mathematics as taught in secondary school: ability to make and read simple drawings, make simple geometric constructions, use right angles, and right triangles; make, read and interpret graphs, and solve problems by the use of ratio. He must also be able to use letters as numbers in simple formulas, and be quick and accurate with fractions, decimals, and per cents in shop situations.

Training for skilled work may be started in the secondary school, but seldom can be finished there. In school the learner becomes accustomed to using simple formulas, performing fundamental operations with algebraic symbols and expressions, learning the significance of positive and negative numbers, using right triangle formulas, and mastering logarithms and their applications. When he enters apprenticeship he is usually given credit for a certain amount of school time against the required number of years.

4. *Farmers*

If a farmer makes a good living on a small farm, he must have a good working knowledge of business arithmetic as offered in the public schools. Such ability concerns computations involving whole numbers, common and decimal fractions, per cents, ratio and proportion. He deals with perimeters, areas, volumes, weights and measures, and must solve problems having to do with liquids (such as milk), solids (vegetables), and gases (fumigants).

Those who attend agricultural colleges are required to take a kind of mathematics course that combines algebra, trigonometry, geometry, and statistics. Some of the topics most often covered in such a course concern: Fundamental operations, fractions, radicals and exponents, linear equations, mixtures (feeds, concrete), ratio and proportion, logarithms, and solution of triangles.

5. *Nurses*

The nurse, usually a woman, needs (1) elementary arithmetic computation, (2) a thorough knowledge of various systems of weights and measures, (3) competence in handling proportions, and (4) skill in reading scales and measuring instruments. She needs to be familiar enough with computations to solve with precision, confidence, and speed the problems that concern the preparing of solutions, food values, calories, dosage, measurements by weights and volume, and changing measurements from one scale to another (avoirdupois, troy, apothecary, and metric weights.) A nurse must be accurate! She does not need higher mathematics.

Summary

The mathematics needed by skilled and semi-skilled workers and by persons in other occupations described in this section is pretty much the same as for personal use. Here also the fundamentals applied to whole numbers, common fractions, decimals, and per cents are very important. Accuracy in trades is perhaps even more important than it is in the mathematics for personal use. You can make mistakes in your personal affairs without losing your job.

In order that too many doors do not close on you early in life, the minimum of high school mathematics for you would seem to be one or two years of general mathematics. A year of mathematics, labelled "shop mathematics" would certainly be good for some pupils. A half year of *statistics* taken in high school would be excellent, but unfortunately it is offered in only a few schools. And as concerns promotion to the more responsible position in the skilled trades the more mathematics the better! This might well take the form of four years of mathematics including *algebra*, *geometry*, and *trigonometry*.

IV. Mathematics for College Preparation

"What mathematics shall I take for college entrance?" That perhaps is the question most often asked by high school students.

The answer depends upon two other questions. (1) What particular college will you attend? (2) What are you planning to do in life?

What College?—You can find colleges that will admit students without any high school mathematics; some are very good ones, including some of our great state universities. But do not be misled by this fact; read carefully the paragraphs that follow. In the second place many institutions will permit you to specialize in a good many fields without having formal algebra, geometry, or trigonometry. For example, you can take a college major in speech, drama, history, languages, political science, and journalism with little or no reference to high school mathematics. There are, however, many doors to opportunities which will be closed to you in college if you have not studied mathemaics in high school.

Furthermore, you can not even enter many professional schools without at least one year in high school algebra and one year in plane geometry. Thus colleges of pharmacy and dentistry require at least one year of algebra and one year of plane geometry in the high school. If you do not take these courses while in high school, you may find that you will be in trouble when you get to college. Colleges are reluctant to offer high school courses for credit although many are forced to do just that. You will have to follow a patchwork program throughout your college years which will keep you out of step with your fellow students in the same field unless you are prepared.

What Specialty?—If you want to be a scientist, say a chemist, you should take all the mathematics given in a good secondary school (algebra, plane and solid geometry, and trigonometry).

Many freshmen after entering college decide that they want to take courses for which they are not prepared in mathematics. They find at least one year of college mathematics is required before they can start their training program for the career they want to follow.

But that is not the end of the story. In certain fields you can take two or three elementary college courses without mathematical training, but the advanced courses become somewhat technical, especially as regards statistics. Typical examples are economics, geography, and sociology.

In any event, your career as a research worker or as a reader of research reports in many many fields as, for example, biology, agriculture, economics, education, sociology, forestry, and business administration, is blocked without a sound foundation in high school mathematics.

The less you are sure of the more you will need to plan. So it might be well to keep the following guides in mind:

1. As early as possible send for the catalog of the college that you have in mind.
2. See what the requirements are, not only for entrance but also of the department concerned. For example the requirements for medicine will probably be different from engineering.
3. If you plan to take one or more courses in college in science or mathematics, then you should probably take four years of algebra, geometry, and trigonometry.
4. If your school has a *good* course in general mathematics, and if such a course is accepted for entrance credit in the college of your choice, the non-science student might well study it instead of the traditional sequence of courses.

V. Mathematics for Professional Workers*

What's a professional mathematician like? What kind of a bird is he? And what does he do?

* See also two bulletins published by the Oklahoma Agricultural and Mechanical College (Stillwater, Oklahoma) entitled (1) Vocational Opportunities for Mathematicians and (2) Mathematics—Its Vocational Aspects.

Roughly speaking, there are two kinds: (1) workers in *pure mathematics*, and (2) workers in *applied mathematics*.

Let's take a look at the *pure mathematician*. There are people who like to play with mathematical laws and principles just to see what will happen. They have mathematical curiosity. They are not especially interested in whether anyone ever finds any use for the result or not. They have the fun of working their problems, and that is the only reward they ask. Believe it or not, there is a large group of men and women getting paid good salaries for having such fun. They are the pure mathematicians. For the most part they are on college and university faculties. Some, however, are employed by business and industrial firms. Out of the play of these pure mathematicians has come some of the most important theories and formulas in modern science. Perhaps you still have the fool notion that the long haired "professor" idles away his time in his Ivory Tower aloof from the world of affairs. Well, that just isn't true anymore—if it ever was. Even if industry or government isn't able to tempt him with a permanent job, he is likely to serve as a consultant time and again on very practical projects.

The second group of professional workers in mathematics are interested in what is known as *applied mathematics*. They work at many problems in business, industry, government, insurance, sociology, psychology, and the sciences. The symbols and laws of mathematics can be used to simplify, and in many cases to make possible, the solution of such problems. In brief, the practical mathematician helps workers in other fields. In fact, he often operates under the name of physicist, chemist, economist, and so on.

Perhaps three illustrations will help make clear what applied mathematics is. (a) Can you see atomic energy in the formula $E=mc^2$? Einstein did! He, along with other mathematicians and scientists, worked with this formula many years before it led them to the secrets of the atomic bomb. (b) Two mathematicians were able to use mathematical formulas to locate accurately a planet that neither they nor anyone else had seen. Lo and behold! The planet was very near where the mathematician said it would be. (c) Mathematicians working with insurance companies use the mathematical laws and formulas of chance to aid them in preparing insurance rates and policies. The number of applied mathematicians employed by both government and industry has increased considerably since the war.

1. *Teachers of Mathematics*

Probably the greatest number of professional workers who use mathematics are employed as teachers. In the following sections you will find descriptions of the mathematical training needed by different types of teachers.

a. *Mathematics for teachers in elementary schools*

There are over a half million teachers in our elementary schools. Not all of them have to teach arithmetic. Some are departmentalized. If you should wish to qualify for such a position, the main requirement would be that you *understand* arithmetic. You cannot teach what you do not know. Here is the arithmetic that you would have to teach in the first six grades.

1. Basic concepts, processes, and vocabulary of arithmetic
2. Our decimal system of numeration, including the concept of decimal fractions
3. Computation, whole numbers and common and decimal fractions
4. Principal units of measurement for everyday use
5. Solution of problems involving computation and units of measurement
6. Identification of geometric figures
7. Use of simple graphs
8. Estimation and checking of answers to problems

In grades 7 and 8 you would teach general mathematics which includes: In-

formal geometry, arithmetic, measurement, graphic representation, arithmetic in personal and community problems, and first steps in algebra.

Many teachers in our elementary schools do not as yet have four years of training in college. The better school systems employ only college graduates.

If you are planning to teach in the elementary school, three things are very important: (1) a good methods course; (2) at least one course which will give you subject-matter background for the mathematics taught in grades one through six; (3) at least one course that will give you subject matter background for teaching the general mathematics of grades seven and eight. Junior high school teachers of mathematics should have at least a minor in college mathematics. This should include a year of general mathematics in college, a course in statistics, a course in the mathematics of investment, and a professionalized course in the subject matter taught in these grades.

Nearly all the teachers in the elementary school are women. However,the elementary school is a grand field for a young man who is a fine person and a competent worker.

b. *Mathematics for teachers in high school*

To an increasing extent high schools are providing a "double track" in mathematics. The high school student may choose either (1) the traditional college preparatory courses, or (2) a series of general mathematics courses.

The traditional college preparatory courses usually begin with elementary algebra (9th grade), followed by plane geometry, and further algebra. Large high schools also provide additional courses that may include trigonometry, solid geometry, college algebra, analytic geometry, and surveying. Of course small schools cannot provide this mathematical feast. However, more and more schools are providing correspondence courses to supplement their offering.

The courses on the "other track" include such electives as (*a*) general mathematics—now widely offered as an alternative to algebra in the ninth grade; (*b*) shop mathematics, and (*c*) for the later years of the senior high school, consumer mathematics and possibly statistics. Students in commercial courses may elect business arithmetic sometimes as early as the ninth grade.

These general mathematics courses are provided for a large number of pupils in the present-day high school who do not desire the college preparatory work or for whom such courses seem unsuited.

Thus it is obvious that in planning your college courses as a future teacher in a high school you should prepare yourself to teach two kinds of courses with very different goals for two different types of high school pupils. It is too bad, but true, that the typical college does not offer too much that is helpful in preparing you for teaching the general mathematics courses that now enroll a majority of the high school pupils in the country.

As a teacher of mathematics in a high school you need not only an aptitude for mathematics, but also those personal qualities necessary for all good teachers. Usually requirements for such jobs include the bachelor's degree, and for large cities, the master's degree with a major in mathematics. It often happens that a teacher must also teach another subject in addition to mathematics.

Positions are open to both men and women who hold teaching certificates. In some states the minimum salary for the beginning high school teacher with four years of college training has been fixed at $2400; and in a few at $2600.

c. *Mathematics for teachers in the junior college*

If you are preparing to teach mathematics in the junior college, you should have one or more years of graduate work and at least a master's degree in mathematics. In addition you should have a substantial preparation in some other field than mathematics, as for example, physi-

cal science, biological science, business, economics, or finance.

d. *Mathematics for the college teacher*

The typical teacher of college mathematics has a doctor's degree (Ph.D.) or is well along toward completing his work for that degree. The courses that you might be called upon to teach are: college algebra, solid geometry, trigonometry, analytical geometry, and calculus. Beyond this point courses branch out into many specialties that concern statistics, finance, pure mathematics, and other phase of work of interest only to those specializing in higher mathematics, or those intending to enter the graduate school.

e. *Mathematics for teachers in the graduate school*

With few exceptions teachers of mathematics in graduate schools have the doctor's (Ph.D) degree. In the United States there are fewer than one hundred schools which award this degree. It may be that you are already thinking about such a school. In that case you will find these institutions listed on page 337.

f. *Mathematics for science teachers*

Mathematics, as you know, is the tool of the sciences. If you plan to be a science teacher, some day you will need about the same kind and amount of high school mathematics as you would if you were going to teach mathematics. With good training in science and mathematics you can shift into either field any time in the first two years of college.

The number of science teachers in the high schools is very large. The total for all sciences is only slightly less than for mathematics.

Administrators of high schools tend to favor men when employing teachers of physics and chemistry. However, a woman who is well prepared will have a choice of positions under present conditions. Salaries of science teachers in high school are about the same as for mathematics teachers.

2. *Statistician*

To be a statistician, first of all, you should be able to pronounce the word! The main thing, however, is to have a thorough knowledge of mathematics. Statistics is not an end in itself. It is a *tool* which is used very widely in modern life.

You have all seen "vital statistics" in the newspapers. At first, statistics served only to provide the public with data relating to births, deaths, exports, imports, and of course census figures.

Government and industry realized however, that these masses of data held information more valuable than just "what." By statistical studies, they could discover the "why" of cycles and trends. Life insurance companies, by using statistical methods, could predict accurately how many people in a particular group would die during a certain year, and they could fix their rates accordingly.

In the last twenty years, statistics as a science has developed into a powerful tool which takes much guess work out of planning and predicting. For example, if the Gallup Poll wants to know how many people are satisfied with their jobs, not every person has to be questioned. A statistical sample rightly chosen will give the answer. Again, if the Navy is buying a huge order of electric light bulbs, not every bulb can be tested. A good statistical sample will be highly satisfactory. Business and economic agencies lean heavily on statistics in purchasing, advertising, and market analysis.

A statistician often plans, directs, coordinates, and conducts statistical research. Poring over and polishing research reports is also a part of his work.

Statisticians work in many fields as for example vital statistics, biometrics, mathematical theory, industrial statistics, educational work, public opinion polls, weather forecasting, and ballistics.

And of course they operate under many titles. Thus the *economic statistician* who collects and interprets data dealing with

money, banking, finance, the cost of living and labor relations may be known as an economist, engineer, or industrialist. The *financial institution statistician* deals with earnings and dividends of corporations, industries, and securities. The *physical science statistician* deals with quantitative problems in exact sciences such as chemistry, physics, mathematics, or biochemistry. The *scientific statistician* works in one of the applied sciences such as chemistry or physics, dealing with qualitative problems. The *social science statistician* handles quantitative problems in economics, sociology, and psychology.

Are you still conscious? Or did these big words stun you? At any rate the graduate study of such fields as economics, sociology, psychology, education requires a great amount of statistics.

Are you aware of the newest and fastest growing application of statistics? Take *quality control*! What's that? In a word it is to control the quality of a product by statistical techniques. You test a pretty small sample and get a fairly reliable rating of the whole.

Some engineers in Bell Telephone had this bright idea about twenty years ago.

During the war both the Army and Navy had to buy or purchase vast amounts of goods. Obviously not every article could be checked. So quality control was introduced widely and rapidly.

In the postwar period large industries are finding out that it pays them to use these statistical techniques.

During the war mathematicians and statisticians and actuaries worked with chemists, physicists, educationists, and psychologists on teams using statistical methods to study men, materials, and processes on the job and in operation. At the rate quality control is growing the idea of a team of scientists and statisticians working on a problem may be common in industry about day after tomorrow.

So fast is the idea of quality control spreading that already the demand for young engineers with the proper statistical training is very great. It wouldn't surprise us if you or some bright classmate would be earning a good salary in this field some day.

Maybe you'd rather work for Uncle Sam. All right, he needs statisticans not only in quality control but also in research projects of great variety and difficulty in such fields as agriculture, education, labor, economics, and the like.

Maybe the salary will look pretty good to you. A statistician's salary in Civil Service ranges from $5905 to $9975 per year. To qualify for this tidy sum you must have ability to (1) plan and direct statistical research projects of great breadth and difficulty, (2) advise government officials on complex administrative or research problems in the field of statistics, (3) prepare the well-written statistical reports of a professional staff. As for background you will need to (1) show a thorough mastery of statistical theory and technique especially as it applies to a specific subject-matter field, (2) carry out difficult professional assignments in statistics with little or no technical supervision, and (3) deal satisfactorily with your fellow workers.

What does all this add up to? A statistician must know his stuff in mathematics! That means taking all of the mathematics offered in high school in preparation for college entrance and as much more as you can tuck in. In addition to college courses in mathematics it is a mighty good thing for the statistician to be trained in some other field in which statistics may be applied; as for example, economics, education, physics. This is putting it midly for training in another field is practically a "must" for anyone who is going to make use of statistics.

It is perhaps needless to say there is a serious shortage of statisticians at the present time. This is largely due to recent growth in statistical projects in govern-

ment, research contracts of colleges and universities, psychological testing, market research, commercial surveys, research in biological sciences, and quality control.

If you like to work with figures, and are doing well in your high school mathematics courses, it is likely that there are very green pastures for you in the field of statistics.

3. *Engineer*

Are you interèsted in engineering as a career? There are many types of engineering activities. Aeronautical, civil, chemical, construction, electrical, industrial, mechanical, metallurgical, and sanitary are among the more important. Some of these areas call for more advanced study of mathematics than do others.

A high school student interested in becoming an engineer should take all the mathematics offered in school—at least two years of algebra, plane geometry, solid geometry, and trigonometry—because he must have the power to think in terms of numbers, formulas, and algebraic symbols. He must then spend at least four years in a college of engineering. In many types of work the engineer should have mathematics through calculus if he expects to be a leader in his profession.

The typical engineering fresman in college according to K. W. Vaughn (*Journal of Engineering Education*, October, 1945) possesses the following characteristics: Male: 17 years of age on last birthday; graduate of a public school of 500 or more students located in the same state as the college attended. His high school mathematics courses included: Algebra (2 years), plane geometry (1 year), solid geometry or plane trigonometry (1 semester). His science courses included General Science (1 year), chemistry or physics (1 year). About half of the students took mechanical drawing for 1 year. Those who avoid such subjects should choose some other career than engineering.

4. *Surveyor*

Surveying is the science of measuring distance. It is a part of the training of every civil engineer. The surveyor measures heights, distances, areas, and locates the position of important points. He uses this information in all forms of construction, map making, land valuation, mining, and many other projects. He is held responsible for the accuracy of his measurements, calculations, and records.

Several semi-skilled workers assist the surveyor. For example, the instrument men adjust the surveying instruments; the chainmen work in pairs measuring distances with a chain; and the rodmen hold the level rods at points indicated by the surveyor.

The 1940 census enumerated 13,243 men and 101 women surveyors. In addition, there were 7,553 men and 19 women who worked as assistants. Finally there are many workers in other fields—forestry service, mining engineering, etc., who need good training in surveying.

The conditions for becoming a licensed surveyor vary with different states. Many states require graduation from an approved school of surveying. Several years practice or apprenticeship is a general requirement.

Preparation for surveying may be begun in high schools that offer short courses for the preparation of helpers in surveying projects and apprenticeship is important. Surveying is taught in any civil engineering college, but courses are not standardized. All of them assume good background in high school mathematics.

Some high schools offer short courses in surveying which may be found very helpful in learning the beginning of the subject. Apprenticeship with an experienced surveyor will also help greatly in learning the trade. The basic mathematics for simple surveying includes geometry, algebra, and trigonometry. In civil engineering, use is made of mathematics through calculus in the study and applications of surveying.

5. *Accountant*

Do not plan to be an accountant unless

you can become agile in tossing figures around. To be sure many accountants gain their skill more by experience than by training. But the flair for figures has to be in the picture at the very beginning.

The accountant, by training, and in practice, is a business analyst. Accurate recording of financial transactions in mathematical terms is essential to the success of any commercial enterprise. The accountant must thoroughly understand "debit and credit" terms generally used by business in its bookkeeping procedure to insure this accuracy. An accountant deals chiefly, except in specialized fields, with every day arithmetical processes, and with percentages, decimals, and ratios.

Some of the problems with which he deals concern: computing simple and compound interest; finding discount and present values; ascertaining funded debt needs; computing sinking fund requirements under actuarial formulas; analyzing installment buying; and dealing with problems of insurance rates and benefits, pension plans, social security taxes, appraisals, and other valuations.

In addition to mathematics, he must be trained in business practice, banking, finance, taxation, economics, corporation law, investments, and insurance coverage. He must relate business enterprise to economic activities. He must understand the principles underlying various accounting records—how controlling accounts function—when adjusting entries are necessary—the differences between partnership and corporate organization—the purpose of depreciation, and the problems of valuation and liquidation. He must be able to prepare periodic balance sheets and profit and loss statements, which are the business charts of financial condition and operating trend.

The qualified accountant of today must understand the principles of budgetary control—how to gather and correlate the data for budgets—how to project financial conditions and operating results——and how to analyze and report on the reasons for differences between projected figures and those which result from operations actually performed.

Income tax laws have given great impetus to accurate accounting under approved and accepted accounting principles. Accounting as it is known and practiced today has made rapid and important strides since the inauguration of the Federal income tax laws on March 1, 1913.

The trained accountant may choose his activity in the field of public accounting where his services and counsel are sought in all phases of accounting practice and complex income tax problems—or he may choose a position as accountant with a single employer in private industry.

Training in accounting is offered in 79 colleges and universities and in many independent private business colleges, as well as through correspondence study.

Since business and industry is getting more and more complicated it isn't at all likely that the supply of good accountants will catch up with the demand in the years just ahead.

6. *Actuary*

Have you ever wondered how insurance companies decide on the rates which they charge for various kinds of insurance? These companies employ *actuaries* who study the data about deaths, accidents, thefts, and all conditions which affect insurance. Then on the basis of this study and by applying mathematics to it, they figure out what rates to charge so that the company can meet all its expenses.

The actuarial field is small compared with other occupational fields, and places of employment are few. The work is so highly specialized that there are only about 300 actuaries in the United States who have met the examinations set for them. It is estimated, however, that at least 1000 could profitably be used in the insurance companies alone.

Concerning examinations, the actuarial societies wield a strong influence in shap-

ing mathematical requirements, and in determining standards for actuarial jobs. They set a group of nine examinations which applicants must pass before they are qualified actuaries. Upon passing all 9 examinations, an applicant becomes a *fellow* of the societies. About 13 fellows are added annually, and in all there are 400 of whom 300 are located in the United States.

The actuarial departments of insurance companies, particularly those located in Connecticut and New York are the largest commercial employers of mathematicians. Actuaries are also employed by such Federal agencies as the Railroad Retirement Board, and Social Security Board. The Civil Service Commission will give information about any openings in actuarial work in the Federal government. Private institutions often employ consulting actuaries in regard to pensions, annuities, service tables, old-age benefits, vital statistics, accident rates, and population studies.

The training required of actuaries is so rugged that those who finally qualify are assured of jobs. Algebraic skills are necessary at all times, and a responsible actuary must know mathematics through algebra, calculus, theory of probability, and the calculus of finite differences, as well as the applications of mathematics to varied problems of life insurance. Few universities offer outstanding training in actuarial mathematics. The larger insurance companies conduct their own training programs to fill the gaps. There are two societies of actuaries in which membership is by examination: (1) Actuarial Society of America, 393 7th Avenue, New York City, and (2) American Institute of Actuaries, 135 S. LaSalle Street, Chicago 3, Illinois.

7. *Medical and Other Health Services*

The group of workers that prevent and cure physical and mental diseases is very large. For example in 1940 there were more than 175,000 physicians, 67,000 dentists, and 100,000 pharmacists.

There are, of course, many other classes in this group as for example dietitians, osteopathic physicians, dental hygienists, occupational therapists, x-ray technicians, physical therapists, professional nurses, optometrists, medical pathologists. The training of all these groups rests very largely on the basic sciences.

If you are planning to enter any one of these fields, you will need a sure foundation in high school mathematics. Moreover, you should not undertake a field like medicine unless you have good ability, rugged health, a flair for the work, a bedside personality, and a great urge to do it, for the training period is long and rigorous.

As regards the future outlook in the medical services, there is convincing evidence of a shortage in the years ahead.

8. *Scientific Researcher*

Chemists, biologists, physicists, astronomers, geologists, metallurgists, economists architects, and other scientists need training in mathematics as well as in their specialties. Many such workers are employed in the government, foundations, industry, insurance companies, engineering firms, and other agencies. These mathematicians and scientists make use of such mathematics as calculus, statistics, differential equations, and vector analysis; and they analyze data in such fields as ballistics, crypt-analysis, fluid dynamics, and the like.

Physicists are required to have a thorough training in advanced mathematics. Demands for men trained in this field are urgent. The work deals with precision instruments, high temperature measurements, automatic electric controls, air conditioning, and the use of new products and materials.

Biologists use mathematical methods especially in physiology, genetics, heredity, medicine, and public health. New fields in biophysics and biochemistry are becoming increasingly important espe-

cially in understanding the nature and effects of radioactivity, X-rays, ultraviolet light, and the electronic microscope.

The demand for scientific workers in the research laboratories in industry has grown by leaps and bounds in the last quarter of a century. For example, a list of such laboratories for 1920 includes 297. The corresponding list for 1940 includes 2264. In 1946 there were about 55,000 scientists in industrial laboratories plus 35,000 technical workers making a total of persons who need good mathematical training of about 90,000. In addition the laboratories in industry employed about 45,000 administrative, clerical, and maintenance personnel. Thus, the grand total of workers in research laboratories and industry in 1946 was around 135,000. The number of professional workers in these research laboratories has grown from about 35,000 in 1940 to 55,000 in 1946.

It may be of interest to you to note the make-up of a research staff and so we provide two illustrations. (*a*) The Dow Chemical Company, Midland, Michigan. The staff includes 650 chemists, chemical engineers, physicists, biologists, mechanical engineers, and metallurgists, and 500 additional personnel to carry on the research work. If you walked in the little town of Midland on Saturday night you might jostle a good many Ph.D's. (*b*) The B. F. Goodrich Company, Akron, Ohio. The staff includes 89 chemists, 15 physicists, 2 biological scientists, 1 metallurgist, 26 engineers, and one mathematician with 109 additional personnel.

There is a very important table just ahead. It not only shows the number of scientific workers in industry at present—57,000—but also the scientific population in Federal government and in the colleges and the universities.

No mathematicians are shown in the table under "Industrial Laboratories." The main reason is that the number is quite small—perhaps 150—but this figure would not tell the story. Few resea ch workers have the title "mathematician" for the reason that when a pure mathematician takes a position in a research laboratory and applies his mathematics to some other field, he usually operates with a title that suggests the field—physicist, engineer, and so on.

Finally, the grand total of about 137,000 is very impressive. For all of these research workers need not only excellent training in mathematics at the high school level, but also a great deal of mathematical training in colleges and universities. Their work requires the highest type of training in applied mathematics and research and this training needs to be combined with creative ability and drive. The mathe-

SCIENTIST POPULATION BY FIELD AND PLACE OF EMPLOYMENT (*Estimate, 1947*)

Field of Work	Federal Government	Colleges and Universities	Industrial Laboratories	Total
Agricultural Sciences	7,900	4,700	—	12,600
Biological Sciences	1,200	5,900	1,750	8,850
Medical Sciences	550	13,500	250	14,300
Physical Sciences	8,000	17,500	27,500	53,000
Physics		1,750	3,900	2,800
Chemistry		2,600	5,800	22,000
Mathematics		750	5,800	—
Other		2,900	2,000	2,700
Engineering	11,400	9,000	21,500	41,900
Miscellaneous	950	—	5,800	6,750
Total	30,000	50,600	56,800	137,400

matician in this research group needs a doctor's (Ph.D.) degree or its equivalent in mathematics.

If you are definitely looking forward to a career in industrial mathematics, you should consult a very helpful pamphlet* by Thornton C. Fry, Mathematical Research Director, Bell Telephone Laboratories. (Readers not especially interested in mathematics may now take a "seventh inning stretch.") Here are some samples of what you will find in this interesting pamphlet:

. . . Industrial mathematics is being carried on by graduates of engineering or physics not so much because of the value of that training as because of the weakness of mathematical education in America. The properly trained industrial mathematician should have, beyond the usual courses of college grade, a good working background of algebra (matrices, tensor theory, etc.), some geometry, particularly the analytic sort, and as much analysis as he can absorb (function theory, theory of differential and integral equations, orthogonal functions, calculus of variations, etc.). These should have been taught with an attitude sympathetic to their applications, and reinforced by theoretical courses in sound, heat, light and electricity, and by heavy emphasis upon mechanics, elasticity, hydrodynamics, thermodynamics and electromagnetic field theory. He should understand what rigor is so that he will not unwittingly indulge in unsound argument, but he should also gain experience in such useful but sometimes treacherous practices as the use of divergent series or the modification of terms in differential equations. He should have enough basic physics and chemistry of the experimental sort to give him a realistic outlook on the power as well as the perils of experimental technique. By the time he has acquired this training he will usually also have acquired a Ph.D. degree, but the degree itself is not now, and is not likely to become, the almost indispensable prerequisite to employment that it is in university life.

There is nowhere in America a school where this training can be acquired. No school has attempted to build a faculty of mathematics with such training in mind. Hence industry has had to make such shift as might be with *ersatz* mathematicians culled from departments of physics and engineering. To make matters worse, a student with strong theoretical interests who enrolls in physics these days is almost certain to spend most of his time on modern mathematical physics, which insists almost as little upon fidelity to experience and experiment as does "pure" mathematics, from which it differs more essentially in matters of language and rigor than of general philosophic attitude. At the moment, therefore, engineering schools must be looked upon as the most hopeful sources of industrial mathematicians.

. . . simple processes as algebra, trigonometry and the elements of calculus are the most common and the most productive in modern industrial research. They frequently lead to results of the greatest practical importance. The single sideband system of carrier transmission, for example, was a mathematical invention. It virtually doubled the number of long distance calls that could be handled simultaneously over a given line. Yet the only mathematics involved in its development was a single trigonometric equation, the formula for the sine of the sum of two angles.

Fry points out that mathematics is a language which simplifies the process of thinking and makes it more reliable. This is its principal service to industry. He then lists eight ways in which mathematics is useful to industry:

First: It provides a basis for interpreting data in terms of a preconceived theory, thus making it possible to draw deductions from them regarding things which could not be observed conveniently, if at all.

An illustration is the standard method for locating faults on telephone lines. Mathematical theory shows that a fault will affect the impedance of the line in a way which varies with frequency, and that the distance from the place of measurement to the fault can be deduced at once from the frequencies at which the impedance is most conspicuously affected. This is obviously much more convenient than hunting the fault directly.

Second: When data are incompatible with the preconceived theory, a mathematical study frequently aids in perfecting the theory itself. The classical illustration in pure science is the discovery of the planet Neptune. The motion of the planet Uranus was found to be inconsistent with the predictions of the Newtonian theory of gravitation, if the solar system consisted only of the seven planets then known. Mathematical investigation indicated, however, that if an eighth planet of a certain size was assumed to be moving in a certain orbit, these discrepancies disappeared. Upon turning a telescope to the spot predicted, the new planet was found.

Third: It is frequently necessary in practice to extrapolate test data from one set of dimensions to a widely different set, and in such cases some sort of mathematical background is almost essential.

An example is provided by Mr. C. T. Reid, Director of Education of the Douglas Aircraft Corporation:

* Reprinted from "Research—A National Resource—II" Section VI, Part 4, pp. 268–288. A House Document, 77th Congress.

"Here it is obviously impracticable to perform full-scale tests of such parts as wings or fuselage, much less of entire aircraft, and the extrapolation from the results of wind-tunnel measurements to the full-scale characteristics of airplanes must be based on theoretical considerations."

Fourth: Mathematics frequently aids in promoting economy either by reducing the amount of experimentation required, or by replacing it entirely. Instances of this kind are met everywhere in industry, not only in research activities, but in perfecting the design of apparatus and in its subsequent manufacture as well.

Fifth: Sometimes experiments are virtually impossible and mathematics must fill the breach. An example comes to me from Mr. Hall C. Hibbard, Vice President and Chief Engineer of the Lockheed Aircraft Corporation:

"An unfortunate phenomenon that must be dealt with in aircraft design is a type of violent vibration which may be set up in the wings if the plane is flown too fast. It is known as flutter, and is highly dangerous, since the vibrations may be of such intense character as to cause loss of control or even structural failure. The technical problem is therefore to be sure that the critical speed at which flutter would occur is higher than any at which the craft would ever be flown. It is a phenomenon with respect to which wind tunnel experimentation is difficult, and flight testing very dangerous. It has been the subject of a number of mathematical investigations, the results of which have reached a sufficiently advanced stage that they are now being used to predict the critical speeds and flutter frequencies of aircraft while still in the design stage. Even more important, the mathematical investigation of this problem points the way to modifications of design which will insure that flutter cannot occur in the usable speed range."

Sixth: Mathematics is frequently useful in devising so-called crucial experiments to distinguish once and for all between rival theories. A famous example in the field of physics was the study of the refraction of starlight near the sun's disc, which afforded a means of deciding between Newtonian and relativistic mechanics. In this case, mathematical investigation showed that the result to be expected was different according to the two theories, and astronomical observations confirmed the prediction of relativistic mechanics.

Seventh: Mathematics also frequently performs a negative service, but one which is sometimes of very great importance, in forestalling the search for the impossible; for many desirable objectives in industry are as unattainable as perpetual motion machines, and frequently the only way to recognize the fact is by means of a mathematical argument.

A certain type of electric wave filter which is usually referred to as an "ideal" filter would be very useful if it could be produced. However, it has been shown mathematically that such a structure would respond to a signal before the signal reached it; in other words, that it would have the gift of prophecy. Since this is absurd, it follows that no such filter can be built, and consequently no one tries to build it.

Eighth: Finally, mathematics frequently plays an important part in reducing complicated theoretical results and complicated methods of calculation to readily available working form. This illustration comes from Mr. Hibbard, Vice-president and Chief Engineer of the Lockheed Aircraft Corporation:

"In aircraft design the metal skin, though thin, contributes a large part of the structural strength. Nevertheless, such thin metallic plates will buckle or wrinkle after a certain critical load is exceeded. Beyond this point the usual structural theories can not be applied directly and it is therefore necessary to introduce new methods of attack to predict the ultimate strength of the structure. These stiffened plates are difficult to deal with theoretically, but by interpreting the effect of the stiffeners as equivalent to an increase in plate thickness or a decrease in plate width, the calculations can be brought within useful bounds."

Mathematics in Some Particular Industries

Perhaps you are even now interested in a particular industry as for example communications, electrical manufacturing, the petroleum industry or aircraft manufacturing. These four industries employ a vast number of technical workers. Let us see what Fry says about the mathematics needed in each of these four industries.

1. *Communications.*—The communications field is the one in which mathematical methods of research have been most freely used. This is due partly to the fact that the transmission of electric waves along wires and through the ether follows laws which are particularly amenable to mathematical study; partly also to the fact that so much of the research has been centralized in a single laboratory, thus bringing together a large number of engineers into a single compact group, and justifying the employment of consultative specialists. Most important of all, however, is the fact that there are two devices—vacuum tubes and electrical networks—without which modern long-distance telephony would be impossible; and one of these, the electrical network, is and has been since its earliest days almost entirely a product

of mathematical research. Mathematics has thus been as essential to the development of nation-wide telephony as copper wire or carbon microphones.

2. *Electrical Manufacturing.*—Substantially all the research in the power fields is carried on by a few electrical manufacturers. The power companies usually accept and exploit such equipment as the manufacturers supply, and contribute to improved design principally through their criticisms of past performance. Many of their engineers, however, are individually active in the invention and development of improved equipment.

3. *The Petroleum Industry.*—The petroleum industry consists of many producing units of various sizes, highly competitive in character, and surrounded by a number of consulting service organizations, all of which are small. The larger producing companies—and within their resources, the service units also—maintain research laboratories.

4. *Aircraft Manufacture.*—The aircraft industry also consists of a number of independent units, and is highly competitive. It is a new industry in which rapid technical development and rapid increase in size has been the rule. It has depended primarily upon government-supported laboratories and, to a lesser extent, upon the universities for its research, and has busied itself with the exploitation of that research in the advancement of aircraft design. No unit of the industry has had or, for that matter, now has a research laboratory, in the sense in which the words would be used in older and larger businesses, but the beginnings of research departments have appeared, and individual researchers and research projects are clearly recognizable.

It isn't likely that there will be an over supply of top flight research scientists in the years ahead. If you can manage to secure a good background of science and mathematics in the high school, you will probably find employment especially with one of the modern industrial corporations that specialize in electrical and scientific appliances. Amazing applications of higher mathematics were made during the war and have since been developed further in such fields as electronics, television, radar, telephonic and telegraphic communication.

9. *Assistants to Research Workers*

You don't have to have four or five years of college training to get a good job in a research laboratory. Many agencies, including the government, employ "assistants" who are trained in special fields and who have a good mathematical background.

These workers operate under many titles as for example, "junior statistician," "scientific computer," "engineer aid," etc. They find employment in particular fields where research projects are being carried on. Good illustrations are engineering, chemistry, geology, mathematics, meteorology, physics, and radio. Much of the work is done in the laboratory or nearby.

As an "assistant" your duties might include adding long columns of figures. But don't get scared at this suggestion. You would probably be doing the elaborate computation with machines. And they might not be ordinary computing machines.

Haven't you been reading in popular magazines about the new giant machines that are a million times faster than the unaided human brain? These monster computers are now being operated, designed, or built in more than fifty laboratories.

It is said that in a few hours these magic computers could solve problems that would keep all mathematicians busy for a hundred years. Some respectable scientists say that these new machines will change life on this planet more than it's ever been changed before largely by controlling and predicting anything that can be measured. As for example weather and economic forces.

As an assistant on some project in industry, insurance, education, or government, you would handle much of the routine work of managing the laboratory materials, collecting and compiling data, and preparing regular reports.

As you would guess varying degrees of training and preparation are necessary. But be clear on this point: these workers must have what mathematical training is offered in high schools—arithmetic, algebra, geometry, and trigonometry.

In addition a major in some field of science is useful. Finally your chances are improved if you have taken some of the following college courses: college algebra, trigonometry, analytic geometry, theory of equations, and determinants. Keep in mind that numerical computation, methods of interpolation, and statistics are likely to be very useful.

VI. Women in Mathematics

Are there only a few girls in your trigonometry and solid geometry classes? Don't be fooled into thinking that there isn't a chance for girls in the field of mathematics.

Before World War II, most girls who majored in mathematics in college became teachers of mathematics. Of course that field is still wide open, and it always will be for strong candidates.

When the war came there was a terrific demand for women trained in mathematics for work in industry, in government, and in research institutions working on government projects. At one women's college every mathematics major had her choice of 25 jobs in industry and government!

Besides demands in teaching and clerical jobs in business firms, there were new demands in computing work in industry and government. Women also found interesting employment in all kinds of engineering work. Those who had combined training in physics with that in mathematics did important research work.

During the war, women with mathematical training who went into military service, did technical work in communications, air navigation, and aerology. Others became computers and cryptographers.

In Federal Civil Service, women trained in mathematics helped win the war by their splendid work in the Ordnance, Signal Corps, and Engineer Corps of the War Department, in the Naval Research Laboratory, the F.B.I., the Bureau of Standards, and the Coast and Geodetic Survey.

You are probably wondering if women are needed in such work now that the war is over. Indeed they are! There is a big field for teaching mathematics in high schools and colleges. The competent woman is sought also for computational and statistical work with insurance and business firms, government agencies such as the Bureau of Standards, and industrial concerns. As stated elsewhere, the field of statistics is growing so fast that there will be a shortage of trained personnel for some time to come. Then too, women with Ph.D's. in mathematics can find positions especially in teachers colleges. Moreover there is at present a rapid expansion of junior colleges.

So girls, if you like mathematics, keep moving! You're on the right track.

VII. Mathematics Used by Civil Service Workers

Any discussion of government work as such would duplicate this entire bulletin, since all types of jobs in mathematics mentioned are available through Federal, State, and local Civil Service. The following discussion is confined to the Federal Civil Service. There is no special preparation for government work in general, but applications for jobs must be in a specific field.

Briefly, the Federal government, through the U. S. Civil Service Commission, Washington, D. C. employs every type of mathematician mentioned in this bulletin—from young clerical workers to experienced professional directors.

In the field of mathematics an individual must first decide upon the nature of work that appeals to him and that he hopes to enter. He then prepares for that specialty in the usual manner, choosing any good approved educational institution for training. National, regional and professional associations accredit or approve schools and colleges; the Federal government does *not* approve or recommend any educational institution.

When a person is prepared in his field, he may wish to learn about openings in the government. The procedure is to write to the Civil Service Commission and request announcements of examinations in his field of interest. These announcements are also posted on the bulletin boards of post offices and other public places. If he chooses to apply for a position, he files the proper application blank, or takes the required examination as directed in the announcement. In this competition he becomes one of many applicants for the job. Finally one individual with the highest qualifications is selected to fill a specific vacancy.

For the lower clerical positions, applicants generally are assembled and given tests which include simple mathematics problems. Those who pass the test are eligible for jobs. Clerks qualifying for the lowest beginning grade that requires mathematical training (CAF-1) enter government employment at a beginning salary of $1756. Examinations are often announced for clerical workers in the four lowest grades where beginning salaries range from $1756 to $2394 a year.

Top grade jobs under Civil Service carry salaries that range up to $9975 a year. In 1947 a few of the professional announcements calling for new workers include:

Mathematician. Salary $7,102 to $9,975 a year—announced January 21, 1947: The education requirement for this position is stated as follows:

> A full 4-year course in a college or university of recognized standing, leading to a bachelor's degree in mathematics. This study must have included courses in mathematics consisting of lectures and recitations totaling at least 24 semester hours, and in addition, courses in the physical sciences (engineering, physics, geology, astronomy, chemistry, etc.) totaling 12 semester hours.
>
> The above courses in mathematics must have included analytic geometry, differential calculus, integral calculus, and in addition any 3 of the following: (*a*) theory of equations, (*b*) vector analysis, (*c*) higher algebra beyond elementary college algebra, (*d*) differential equations, (*e*) advanced differential calculus, (*f*) advanced integral calculus.

Although no written examination was required, applicants were rated on a basis of their education and experience as shown on their application blanks.

Accountant. Salary $4,149 to $5,905 a year—announced Feb. 11, 1947.

This announcement calls for four types of accountants all for the Securities and Exchange Commission; Research Accountant, Office of the Chief Accountant; Financial Accountant, Corporation Finance Division; Public Utilities Accountant, Public Utilities Division; Securities Accountant, Trading and Exchange Division and Field Offices.

Experienced workers are sought, but applicants may substitute education (1 year of study for 9 months' experience) for not more than 3 years of experience under certain conditions. For example: Study in accounting, finance, business administration, or banking successfully completed in a college or university of recognized standing, or at a residence school above high school level.

Statistician. $5,905 to $9,975 a year—announced February 11, 1947 has been discussed under the topic "statistician."

IX. Mathematical Organizations

Actuarial Society of America, 393 7th Ave., New York, N. Y.

American Institue of Actuaries, 135 S. LaSalle St., Chicago 3, Ill.

American Mathematical Society, concerned with research in mathematics, 531 West 116th St., New York 27, N. Y.

Econometric Society, interested in the

VIII. Graduate Schools Offering Doctorates in Mathematics

Note: Figures indicate number of Ph.D. degrees awarded from 1930 to 1939.

Calif.	Berkeley	Univ. of California	27
	Pasadena	California Institute of Technology	15
	Stanford	Stanford University	7
Colo.	Boulder	Univ. of Colorado	3
Conn.	New Haven	Yale University	20
D. C.	Washington	Catholic University	15
	Washington	George Washington University	2
Ill.	Chicago	Northwestern	2
	Chicago	Univ. of Chicago	94
	Urbana	Univ. of Illinois	54
Iowa	Ames	Iowa State College	5
	Iowa City	State Univ. of Iowa	19
Ind.	Bloomington	Indiana University	6
	Notre Dame	University of Notre Dame	1
	Lafayette	Purdue University	1
Kan.	Lawrence	Univ. of Kansas	1
Ky.	Lexington	Univ. of Kentucky	4
La.	Baton Rouge	Univ. of Louisiana	1
Md.	Baltimore	Johns Hopkins University	18
	College Park	University of Maryland	1
Mass.	Cambridge	Harvard University	43
	Cambridge	Mass. Institute of Technology	27
	Cambridge	Radcliffe College (women)	6
Mich.	Ann Arbor	Univ. of Michigan	60
Minn.	Minneapolis	Univ. of Minnesota	8
Mo.	Columbia	Univ. of Missouri	9
	St. Louis	St. Louis University	3
	St. Louis	Washington University	6
Neb.	Lincoln	Univ. of Nebraska	3
N. J.	Princeton	Princeton University	37
N. Y.	Ithaca	Cornell University	37
	New York	Columbia University	21
	Troy	Rensselaer Polytechnique Institute	5
	New York	New York University	11
N. C.	Durham	Duke University	7
	Chapel Hill	University of North Carolina	5
Ohio	Cincinnati	Univ. of Cincinnati	13
	Columbus	Ohio State University	29
Pa.	Philadelphia	Bryn Mawr (women)	8
	Philadelphia	University of Pennsylvania	18
	Pittsburgh	University of Pittsburgh	18
R. I.	Providence	Brown University	25
Tenn.	Nashville	George Peabody College	4
Tex.	Austin	University of Texas	9
	Houston	Rice Institute	11
Va.	Charlottesville	University of Virginia	7
Wash.	Seattle	University of Washington	4
W. Va.	Morgantown	Univ. of West Virginia	2
Wis.	Madison	University of Wisconsin	30
	Milwaukee	Marquette University	2

mathematical theory of economics, University of Chicago, Chicago 37, Ill.

Central Association of Science and Mathematics Teachers, Inc., P.O. Box 408, Oak Park, Ill.

Institute of Mathematical Statistics, concerned with the mathematical theory of statistics, 116 Rackham Hall, University of Michigan, Ann Arbor, Michigan.

Mathematical Association of America, devoted to the interests of collegiate mathematics, University of Buffalo, Buffalo 14, N. Y.

National Council of Teachers of Mathematics, concerned with problems of mathematics education, 525 W. 120th St., New York, N. Y.

X. Selected References

Books and Pamphlets

The Outlook for Women in Mathematics and Statistics. U. S. Department of Labor, Women's Bureau.

Personnel and Training Problems Created by the

Recent Growth of Applied Statistics in the United States, National Research Council, 2101 Constitution Ave., Washington, D. C. (price 25¢).

"Qualifications for Professional Positions in Applied Mathematics and Related Scientific Fields." R. S. Burlington and D. C. May, *Journal of Engineering Education*, Vol. 37, No. 8, April, 1947.

Statistical Methods for Research Workers. R. A. Fisher.

Careers for Women in Banking and Finance. Dorcas Campbell, N. Y., Dutton & Co. 1944.

Bookkeeping, Success Vocational Information Series, No. 20. Chicago, Morgan-Dillon & Co. 1937, 24 p. 32¢.

Actuarial Examinations, Preliminary. Actuarial Society of America, 393 7th Ave., N. Y., I, N. Y. and American Institute of Actuaries, 135 So. LaSalle St., Chicago, Ill. 1946–47, 26 p. Free.

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Clerical Occupations. Schoerb and Medskr. Chicago, Science Research Associates, 228 So. Wabash Ave., 1940, 48 p. 50¢.

Job Descriptions for the Construction Industry. U. S. Employment Service. Washington, Gov't. Printing Office. July, 1936. 5 vols., 1539 p. Sold in sets only, $5.75.

National Apprenticeship Program. War Manpower Commission, Washington 25, D.C. 1944, 8 p. Free

Accountancy as a Career Field. Geo. A. Spaulding. Washington, D. C. National Council of Business Schools. 1944, 14 p. 10¢.

Engineers' Council for Professional Development, Annual Report. Engineering Societies Bldg. 25 W. 39th, N. Y. 26 p. 25¢.

Careers in Science. Phillip Pollack. New York, Dutton & Co., 1945, 222 p. $2.75.

Opportunities for Statistical Workers. Donald E. V. Henderson. Chicago, Science Research Associates. 1938, 58 p. 60¢.

Periodicals

School Science and Mathematics. P.O. Box 408, Oak Park, Ill. 9 copies a year.

Scripta Mathematica. Amsterdam Ave. & 195th, N. Y., N. Y. Quarterly.

The Mathematics Teacher. 525 W. 120th, N. Y., N. Y. 8 copies a year.

Journal of the American Statistical Association. Menasha, Wis. Quarterly.

In the March 1945 number see the following articles:

Role of the American Statistical Assoc. Helen M. Walker.

Opportunities for statistical work in the Federal Gov't. W. R. Leonard.

Opportunities for statistical work in State local Govt's. David M. Schneider.

Opportunities for teaching and research. Gertrude M. Cox.

Opportunities for statisticians in industry. John B. Catlin.

In the June 1945 number see the following articles:

Training at the professional level for statistical work in agriculture and biology. W. G. Cochran.

On the training of statisticians. Milton da Silva Rogregues.

American Mathematical Monthly. Oberlin, Ohio. 10 issues each year.

Bulletin of The American Mathematical Society, 12 issues each year.

Mathematics Magazine (Formerly *National Mathematics Magazine*).

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The Commission is especially indebted to Ruthann Bales Schorling who read the manuscript with youthful eyes; and to Miss Veryl Schult, supervisor of mathematics, Washington, D. C., who gave her time and energy unstintingly in aiding the editorial committee. Then too, the Commission is greatly obligated to Harry Jager, Chief, Occupational Information and Guidance Service, Federal Security Agency, U. S. Office of Education, for his generous cooperation.

Finally, this report might never have been published without the help of Walter J. Greenleaf, who has written a goodly number of guidance leaflets for the U. S.

Office of Education. It was he who organized the materials submitted by many persons in a form appropriate to a guidance pamphlet and wrote the first over-all draft of this report. However, he should not be held responsible for the shortcomings of this report for the reason that the editorial committee of the Commission rewrote many of the parts in the effort to gear the vocabulary and the ideas to the high school level.

The Commission has leaned over backwards in its effort to get a guidance report free from unfair propaganda for mathematics as a school subject. To that end we submitted this report to the following curriculum specialists for criticisms and comments: Thomas H. Briggs, H. R. Douglass, J. Paul Leonard, and J. Cecil Parker.

These quotations will give the reader a general picture of their reactions:

Thomas H. Briggs:

I think the guidance report is a very good job, contributing needed information to young people.

Harl R. Douglass:

I have looked over your "Guidance Leaflet—Mathematics," and I think it is splendid. It has avoided all of those wild, unsupportable claims for the values of mathematics which teachers use to recruit students for their classes.

J. Paul Leonard:

I am sure any high school boy or girl would profit greatly from reading it and I am sure it would give most of them a new idea of the importance of mathematics to certain jobs. It is attractively written and is remarkably authoritative and complete to be so concise. I recommend it strongly to high school youth and to guidance counselors.

J. Cecil Parker:

The guidance report is a good job and will be genuinely useful to teachers, counselors, pupils and parents. It should help to meet a real need for sound information upon which to base choices of courses and experience in high school.

Raleigh Schorling (Chairman), University of Michigan; William Betz, Public Schools, Rochester, New York; William A. Brownell, Duke University; Walter H. Carnahan, Purdue University; Eugenie C. Hausle, Public Schools, New York City; Virgil S. Mallory, Montclair Teachers College; C. V. Newsom, Oberlin College; Mary Potter, Public Schools, Racine, Wisconsin; H. Vernon Price, University High School, Iowa City, Iowa; William L. Schaaf, Brooklyn College; Rolland R. Smith, Public Schools, Springfield, Massachusetts; Ruth Sumner, Public Schools, Oakland, California; F. Lynwood Wren, George Peabody College For Teachers; James H. Zant, Oklahoma A and M College.

A Device for Teaching the Extraction of Roots

By P. A. Sample
Anderson, California

The following is an exposition of a formulary procedure for the extraction of roots of the *nth* degree, where n is any positive integer. While this procedure may not have great practical value beyond its application to square and cube roots, standard categorical rules for which are to be found in most texts of advanced arithmetic and algebra, it is nevertheless the belief of the author of this article that all students of pure, as contradistinguished from applied, mathematics will be interested in this exposition, since it is but a logical extension and application of a universally known and used algebraic formula—the Binomial Theorem.

Moreover, it is believed that the procedure herein set forth will be found by teachers of advanced arithmetic and algebra to have unusual value in connection with their instruction of pupils in square root and cube root. It is a notable, if not a notorious, fact that, excepting the relatively few pupils who elect mathematics as a college major and spend years in such courses, practically no one remembers how to "do" square root and cube root beyond the end of the semester in which he receives his instruction in those processes. However, it has been the experience of the author of this article that no student who has once been thoroughly instructed in the procedure detailed below has ever forgotten how to extract square root and cube root.

The reason why practically any pupil, who masters the procedure of root extraction by the method detailed below, acquires a fixed knowledge and permanent remembrance of the process seems to be the high degree of imaginative enthusiasm to which he attains when he discovers the ease with which he accomplishes the extraction of roots of high indices, as, for instance, the 5th, 7th, 11th, etc. roots. The technique of instruction, therefore, which the author suggests is as follows: Having laid the foundation for the procedure by making sure that the pupil has mastered the relatively simple expansion of a binomial in accordance with the Binomial Theorem, let the instructor proceed at once to demonstrate the method by the extraction of, say, the 5th or 7th root of a selected number which is the exact power, 5th or 7th, of the root which he expects to obtain in his demonstration. Now he is ready to have his pupils, using the demonstrated process on other selected numbers, extract roots of the same index. Then, with still other carefully selected numbers, let the drill proceed, using numbers of descending powers for the extraction of roots of corresponding descending indices. For example: If the demonstration and drill began with, say, the 7th root of a number, then the application of the procedure to the 6th root, the 5th, the 4th, the 3d (cube), and the 2d (square) should be proven. The experience of this author has been that almost without exception his pupils have declared, when square and cube root have been reached, such roots are so easy to obtain that they could almost "do them in their heads." This, of course, was the main objective of the instruction from the beginning: the overlearning of the process in its application to square and cube roots, so that the pupil would not forget it. And it has worked.

As a preliminary to instruction in the recommended procedure which is outlined below, it is presumed, therefore, that the relatively simple expansion of a binomial in accordance with the Binomial Theorem has been mastered by the pupil.

First, then, set down the expansion of $x+y$ to the n power, n being the radical index of the root to be extracted—for

example: the square of $x+y$ for the square root; the cube, for cube root; the fourth power, for fourth root; the fifth power for fifth root; etc.

Next, excluding the first term, factor as a polynomial all remaining terms of the expanded expression, setting y—the removed factor—outside parentheses. Illustration: $x^2+2xy+y^2$, second power of $x+y$; $x^3+3x^2y+3xy^2+y^3$, third power of $x+y$; and $x^4+4x^3y+6x^2y^2+4xy^3+y^4$, fourth power of $x+y$, would appear as follows when factored: $x^2+(2x+y)y$, $x^3+(3x^2+3xy+y^2)y$, and $x^4+(4x^3+6x^2y+4xy^2+y^3)y$. Thus, it will be seen, in each case the expanded polynomial expression has been changed into an equivalent complex binomial.

Now, underneath the factored binomial expression, write the number whose root is to be extracted, being careful to separate it into periods, with n digits to each period; i.e., two digits for square root, three digits for cube root, five digits for fifth root, etc. This periodizing procedure must start at the decimal point and proceed towards the left if the given number is a whole number; towards the right if it is a decimal; and in both directions, if it is a composite number.

Two observations at this point should be borne in mind: 1st, all whole-number periods must contain n digits, except the one at the extreme left, which may contain as few as one digit; and 2d, *all* decimal periods must contain n digits. If the given decimal, when separated, should be found to have less than n digits in the extreme right period, enough ciphers must be annexed to it to complete its complement of n digits. A further detail regarding the periodizing of a decimal might well be added here. If more decimal places are desired in the root than are indicated by the periods shown in the given number, additional periods composed of ciphers should be annexed at the right of the decimal to make up the number of places desired in the root. This is necessary—and of course apparent—when the root of a small number which is an imperfect power is desired. For example: if the cube root of the number 2 is desired, correct to four decimal places, four periods of three ciphers each, or twelve ciphers, should be annexed to the number at the right of the decimal point.

Now the actual operation of the extraction of the *nth* root begins. Starting with the first period at the extreme left of the number, find that number between zero and ten which, raised to the *nth* power, most nearly approximates, but does not exceed, the number represented by the first (extreme left) period. Set this number down as the first (extreme left) digit of the required root. Then place the *nth* power of this number underneath the first period number and subtract. Now bring down the next period and annex it to the remainder, if any, resulting from the subtraction. This constitutes the dividend for the determination of the second digit of the required root.

This number is determined in the following manner. Take the factored portion of the expanded power of $x+y$ as a guide for the setting up of the divisor for the indicated dividend. This divisor, when found, will be the sum of as many different numbers as there are terms enclosed within the parentheses of the factored binomial expansion, or n numbers. Write down these numbers, as they are determined, for addition, at the left of the dividend. Taking the first term within the parentheses, (which is always nx^{n-1}, x being throughout the entire operation, and repeatedly, the total number represented by the digit or digits at the moment in the developing root), raise this x number to the $n-1$ power, then multiply this power product by n, the coefficient of x, with as many ciphers annexed to it as there are remaining terms in the parentheses, or as is indicated by the exponent of x in the expression. (See footnote) In the case of square root this will be 2 with one cipher annexed, or 20; of the cube root, 3 with two ciphers, or 300; of the 4th root, 4 with

three ciphers, or 4000; of the 5th root, 5 with four ciphers, or 50,000; etc. Use this completed product as a tentative trial divisor to find the next digit of the root. This will be y wherever it occurs in the parenthetical portion of the expanded and factored binomial. Then complete the divisor by taking each succeeding term within the parentheses, raising the x portion of the root to the indicated power, multiplying this result by the coefficient of x with as many ciphers annexed to it as there are remaining terms in the parentheses, or as is indicated by the exponent of x in the expression (see footnote), then multiplying this product by y raised to the indicated power in the term which is being developed. The last term in the parentheses, representing the last number in the sum to make up the complete divisor, will be y^{n-1}, which will be the last digit just set down in the root, raised to the $n-1$ power. The sum of these several products is the complete divisor.

Of the factored portion of the expanded binomial, only y outside the parentheses remains to be taken care of. This, which stands for the last digit set down in the root, is the coefficient of the parentheses, and as such, of the completed divisor. Multiply the divisor by this number, set the product underneath the dividend, subtract, bring down the next period, and follow the identical procedure until all the periods of the number under the radical have been used. The resulting number is the required root.

For purposes of teaching, several features of this procedure, though implied in the above exposition, might profitably be emphasized by calling the pupil's attention to them in connection with the examples set down below, or similar ones.

1. There are always n terms in the parentheses after factoring the binomial expansion. Consequently, there will always be n numbers to be added to make up the complete divisor at each step in the process of root extraction.

2. The first term of the parentheses contains only x to the $n-1$ power with its coefficient of n. The last term consists only of y to the $n-1$ power, and its coefficient is always 1.

3. The intermediate terms, if any, all have coefficients equal to, or greater than, n; x beginning with an exponent of $n-1$ and following an order of descending powers, y beginning with an exponent of 1 and following an ascending order.

4. X is always the total number represented by the partial root at the close of the previous period operation, and may be a number of one or several digits, as the case may be. (See footnote)

5. Y is always a unit digit between zero and nine, inclusive, and is the resultant quotient of the division in the immediate period's operation.

NOTE: Teachers of mathematics and mathematicians in general will note particularly that the title of this article is "A DEVICE for Teaching the Extraction of Roots." The word, Device, is used advisedly; for, what actually occurs in the procedure of setting up each complete divisor for its related period is, from the standpoint of pure mathematics, something slightly different, though in every instance the numerical result is the same. Since our system of notation is the decimal system, any number at all above nine may be considered an expression of tens plus a unit between 9 and 0 inclusive. For instance: 47 is 40 or 4 tens plus 7; 269 is 260 or 26 tens plus 9, etc. Accordingly, any positive number may be represented algebraically by the binomial $x+y$, x representing the tens and y the units of the number. For example: in the numbers given above x is 40 and y is 7; also x is 260 and y is 9. Therefore it will be apparent at once that in the development of any product of x and y, x is actually a statement of tens and y some unit less than ten. Moreover, the product of x and y may have any coefficient, and both x and y may each have any exponent; yet the

principle holds good. Furthermore, it will be seen that it makes no difference numerically whether x as a unitary expression be raised to its indicated power and then be multiplied by its coefficient with the power-indicated zeros affixed, or whether it (x) be taken as an expression of tens, as such be raised to the indicated power, and then be multiplied by its simple coefficient as indicated in the given expression. For example: take the expression $6x^2y$ where $x+y$ is $320+5$ or 32 tens$+5$. Then $6x^2y=3072000$. This result may be obtained by raising 320 to the 2d power, multiplying it by 5, and then by 6. Also the same result will be obtained if 32 be squared, then multiplied by 5 and then multiplied by 6 with 2 ciphers affixed, or 600. As a procedure for use in the instruction of the average pupil, the author has found the latter—as a device—easier to fix in the mind of the pupil; hence this procedure has been set forth at length in the body of this article.

Below are examples of the suggested procedure:

SQUARE ROOT

$$(x+y)^2=x^2+2xy+y^2$$
$$=x^2+(2x+y)y$$

					6	4	1 7
				$\sqrt{41}$	17	78	89
x^2	=	6^2	=	36			
$2x$	=	6×20	=	120	5	17	
y	=			4			
$2x+y$	=			124			
$(2x+y)y$	=	$124\times$	4 =		4	96	
$2x$	=	64×20		1280	21	78	
y	=			1			
$2x+y$	=			1281			
$(2x+y)y$	=	$1281\times$	1 =		12	81	
$2x$	=	641×20	=	12820	8	97	89
y	=			7			
$2x+y$	=			12827			
$(2x+y)y$	=	$12827\times$	7 =		8	97	89.

CUBE ROOT

$$(x+y)^3=x^3+3x^2y+3xy^2+y^3$$
$$=x^3+(3x^2+3xy+y^2)y$$

					1	0	3	2
				$\sqrt[3]{1}$	099	104	768	
$x^3=1^3$	=			1				
$3x^2=1^2\times300$	=			300	99			
$3x^2=10^2\times300$	=			30000	99	104		
$3xy=10\times30\times3$	=			900				
$y^2=3^2$	=			9				
$3x^2+3xy+y^2$	=			30909				
$(3x^2+3xy+y^2)y$	$=30909\times3$	=			92	727		
$3x^2=103^2\times300$	=			3182700	6	377	768	
$3xy=103\times30\times2$	=			6180				
$y^2=2^2$	=			4				
$3x^2+3xy+y^2$	=			3188884				
$(3x^2+3xy+y^2)y$	$=3188884\times2$	=			6	377	768.	

FOURTH ROOT

$$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$$
$$= x^4+(4x^3+6x^2y+4xy^2+y^3)y$$

					3	2	5
					$\sqrt[4]{111}$	5664	0625
$x^4 = 3^4$	=			81			
$4x^3 = 3^3\times4000$	=		108000	30	5664		
$6x^2y = 3^2\times600\times2$	=		10800				
$4xy^2 = 3\times40\times2^2$	=		480				
$y^3 = 2^3$	=		8				
$4x^3+6x^2y+4xy^2+y^3$	=		119288				
$(4x^3+6x^2y+4xy^2+y^3)y$	$=119288\times2$	=		23	8576		
$4x^3 = 32^3\times4000$	=		131072000	6	7088	0625	
$6x^2y = 32^2\times600\times5$	=		3072000				
$4xy^2 = 32\times40\times5^2$	=		32000				
$y^3 = 5^3$	=		125				
$4x^3+6x^2y+4xy^2+y^3$	=		134176125				
$(4x^3+6x^2y+4xy^2+y^3)y$	$=134176125\times5=$			6	7088	0625.	

FIFTH ROOT

$$(x+y)^5 = x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$$
$$= x^5+(5x^4+10x^3y+10x^2y^2+5xy^3+y^4)y$$

					2	2	2
					$\sqrt[5]{53}$	92186	09632
$x^5 = 2^5$	=			32			
$5x^4 = 2^4\times50000$	=		800000	21	92186		
$10x^3y = 2^3\times10000\times2$	=		160000				
$10x^2y^2 = 2^2\times1000\times2^2$	=		16000				
$5xy^3 = 2\times50\times2^3$	=		800				
$y^4 = 2^4$	=		16				
$5x^4+10x^3y+10x^2y^2+5xy^3+y^4$	=		976816				
$(5x^4+10x^3y+10x^2y^2+5xy^3+y^4)y$	$=976816\times2$	=		19	53632		
$5x^4 = 22^4\times50000$	=		11712800000	2	38554	09632	
$10x^3y = 22^3\times10000\times2$	=		212960000				
$10x^2y^2 = 22^2\times1000\times2^2$	=		1936000				
$5xy^3 = 22\times50\times2^3$	=		8800				
$y^4 = 2^4$	=		16				
$5x^4+10x^3y+10x^2y^2+5xy^3+y^4$	=		11927704816				
$(5x^4+10x^3y+10x^2y^2+5xy^3+y^4)y$	$=11927704816\times2=$			2	38554	09632.	

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A Study of the Process of Transformation with Special Emphasis on Its Application to the Division of Decimal Numbers

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Recently several interesting articles on the division of decimal numbers appeared in The Mathematics Teacher,[1] and in various other professional publications referred to in this paper. When a subject is thus widely discussed, it points to a healthy quest on the part of teachers to get at the root of the matter.

It is almost a truism to say that one's philosophy influences his educational goal and his means of attaining such goal. Consequently the findings herein stated will be readily accepted by those who hold that the purposes, meanings, and relationships of arithmetic processes must be known before they can be successfully applied to the solution of social situations. The teacher trains the child to understand the science of arithmetic, and guides him in building up and applying its principles to social situations presented either tangibly or by written problems.

It should here be emphasized that arithmetic is an abstract instrument invented to aid the mind in building up and controlling social institutions that meet man's present and future needs. Government, education, religion, recreation, production, manufacturing, distribution, banking, communication, and transportation are all institutions that were created to satisfy man's desires. To understand their purposes and the social situations which their activities create, a knowledge of the principles of arithmetic is required.

It has become almost trite to say that children have not been taught until they understand. Every time a child memorizes a rule or mechanically manipulates a procedure without understanding what he does or why he does it, he erects an interference factor to thinking. Memory and thinking are not synonymous, but reasoning in arithmetic builds up power to think in number situations and the first purpose of education is to teach our young people to think. Memorizing arithmetic processes does not guarantee their retention or their successful application. The findings of recent surveys support this view.[2]

In the first three articles referred to herein, two methods, the caret and the inverse, for teaching the division of decimals were discussed. The inverse method seemed to be preferred by the writers of the last two articles, who believe it to be more meaningful and less of a mechanical device than the caret method. They claim this to be true since it is the inverse of the multiplication of decimals in which both multiplier and multiplicand are treated as integers during computation, after which a number of decimal places, equal to those in both the multiplier and multiplicand, are counted off in the product. Since the caret method is admittedly a mechanical device, let us examine the inverse, or subtraction method, to see whether it is more meaningful. Those who defend it seem to overlook the fact that treating both multiplier and multiplicand as integers, and the counting off of decimal places in the product, are also mechanical devices. This may be shown by computing the following example as if the multiplier and multiplicand were integers, and then counting off

[1] *Some Thoughts on Placing the Decimal Point in Quotients.* Claude H. Brown, Feb., 1945, p. 78. *Some More Thoughts on Placing the Decimal Point in Quotients.* J. T. Johnson, May, 1945, p. 229. *Division of Decimal Fractions.* Mary E. Crofts, Apr., 1946, p. 178.

[2] *Difficulties in Decimals Encountered by Ninth Grade Pupils.* Walter Scribner Guiler, Elem. School Journal, March, 1946, p. 384.

the decimal places in the product according to the rule:

$$\begin{array}{r} .2\ 5 \\ 3.7 \\ \hline 175 \\ 75 \\ \hline .925 \end{array}$$

Everything that was done in the above computation is in violation of the principles underlying the decimal system of notation and contrary to what has been taught to children up to this point. In decimal notation all decimal points should be under each other so as to keep the tens, ones, tenths, hundredths, etc., in their places. By what method of reasoning may we ignore the decimal point and consider both multiplier and multiplicand as integers when the meaning and purpose of the decimal point is the very thing we are trying to teach. This must be so because unlike units cannot be added or subtracted. If we examine the notation of the above example we find that thousandths are written under tenths and tenths under ones. In other words, the decimal point keeps on jumping. When will we be able to fasten it down and make it stay "put?" It can be done when we remain logical and consistant with the decimal system of notation,[3] as:

$$\begin{array}{r} .25 \\ 3.7 \\ \hline .175 \\ .75 \\ \hline .925 \end{array}$$

Explanation:

a. 7 tenths $\times$ 5 hundredths = 35 thousands = 3 hundredths and 5 thousands over

b. 7 tenths $\times$ 2 tenths = 14 hundredths + 3 hundredths = 17 hundredths = 1 tenth and 7 hundredths over

c. 3 ones $\times$ 5 bundredths = 15 hundredths = 1 tenth and 5 hundredths over

d. 3 ones $\times$ 2 tenths = 6 tenths + 1 tenth = 7 tenths.

Adding:

a. 5 thousandths

b. 12 hundredths = 1 tenth and 2 hundredths over

c. 8 tenths + 1 tenth = 9 tenths

d. together, 9 tenths, 2 hundredths, and 5 thousandths, or .925.

There is then no need for counting off decimal places mechanically when multiplication is intelligent thinking rather than a memorized device. Dvision of decimals can likewise be done in a meaningful manner, as we shall see later.

Probably the error currently made in teaching arithmetic lies in our assumption that concepts of mathematics needed in arithmetic can be taught in the elementary school by bringing them down to the children from above, rather than by building them up from below and letting children recreate them for themselves. We begin with an overcrowded curriculum heavy with ready made shortcuts and rules invented by a few brilliant adult minds and try to pass them down to children who have difficulty with abstractions and who learn best by objective demonstration and in tangible situations consonant with their maturity level. The mathematics teacher should be able to start with children from where he finds them instead of trying to pull them up to his own level. This means that basic insights, meanings, relationships, and skills begun in the elementary school should not be discontinued on the secondary level. Probably this would improve the arithmetic results now obtained by high school and university students. If children were taught objectively throughout the grades, aided by a study of the historical development of arithmetic concepts,[4] they could be made to see meanings and purpose in the arithmetic they are to learn.

Primitive man gradually mastered his environment and improved it in order to

[3] *Streamlining Arithmetic.* Louis E. Ulrich, Sr., Lyons & Carnahan. 1943, pp. 147–154.

[4] *Streamlining Arithmetic.* Louis E. Ulrich, Sr., Lyons & Carnahan. 1943, chapter 4.

provide for his expanding needs. As his mental powers increased, the difficulty of subjugating the tangible things about him decreased. As his thinking became more abstract he was able to conceive a greater number of objects he sought to produce and control, and to free himself from their immediacy. This remote control and the power to produce in abundance could have been accomplished more readily had it not been for the fact that man has two very definite mental limitations. The first of these is his innate difficulty to conceive more than five discrete objects or units in isolation. The second is his incapacity to employ with facility for computative purposes more than one specific kind of unit of measure, or means of notation at one and the same time.

His inability to conceive more than about five units in isolation is shown by primitive man's tendency to match with the five fingers on each hand, or by our present use of tally marks for scoring points in a game or votes at an election, as: 𝍸, 𝍸 11, for 12. In a recent book,[5] Henry P. Van Dusen quotes from the records of Dr. George Brown, a pioneer missionary to New Ireland: "On the trunk of the palm just outside, seventy-six notches (tallies) record past orgies," showing that these natives were unable to count. In the same book Mr. Van Dusen tells how Dr. E. E. Bromilow in New Guinea, with the assistance of a native teacher, Eliesa, translated the Bible into one of the native dialects, and "numbers above five had to be created." The smaller the groups of units the easier it is to conceptualize them. Thus seven is easily conceived as ⁘ ∴, and nine as ⁘ ⁘ ∴. By objectifying small groups or combinations of them the teacher is provided with an effective method of teaching the concepts to ten.

Man, however, had to develop concepts of sums greater than ten. He therefore gradually invented counting.[6] This was done by transforming each ten units, equal to the number of fingers on both hands, to one larger unit, ten, so that when he had matched ten objects with his fingers, he conceived and represented them as 10, meaning literally not ten individual units, but one ten and no ones. Thus 75 means seventy-five ones matched and seventy ones transformed to 7 tens, with 5 ones left over. We are now able to conceive 75 because we have, instead of seventy-five individual units, only twelve units, namely, 7 tens and 5 ones, as: ⁘ ⁘ tens and ⁘ ones. By counting we have thus transformed many small units to a few larger ones, small enough in number so that the mind can grasp their frequency. Numbers are thus built up by counting. Any number may be defined as the sum of the products of the frequencies by the different sized units which compose it, as: 253.76 means

frequency	unit	product
2	100.	200.
5	10.	50.
3	1.	3.
7	.1	.70
6	.01	.06
		253.76 Sum.

Addition and multiplication are shortcuts for counting in that many units are likewise transformed to fewer larger ones. Thus $75+30+95=200$, and $8\times25=200$, that is, no ones, no tens, but 2 hundreds. We have in this manner overcome our natural inability to conceive many discrete units by the invention of the abstract instrument we call counting.

In order to clarify the meanings of numbers as used in this study let us define an integer as a decimal number composed of frequencies of decimal units greater than tenths, a decimal fraction as a decimal number composed of units smaller than ones, and a mixed decimal as a num-

[5] *They Found The Church There. The Armed Forces Discover Christian Missions.* Henry P. Van Dusen. Friendship Press. New York, 1945, pp. 16 and 20.

[6] *Arithmetic Terminology.* Louis E. Ulrich, Sr., Wisconsin Journal of Education, March, 1945, p. 32.

ber composed of frequencies of decimal units greater than tenths and smaller than ones. Thus all are decimal numbers. The term "fraction" is here used to express both ratio and a frequency of units smaller than ones.

This process of transforming many smaller units to fewer larger ones is done in reverse when finding the difference, or degree of difference (ratio) between numbers, as in subtraction and division. In finding the difference between 75¢ and 7¢ the notation is as follows:

$$\begin{array}{r} 75¢ \\ -7¢ \\ \hline \end{array} \qquad \text{or} \qquad \begin{array}{r} \$\ .75 \\ -.07 \\ \hline \end{array}$$

1 ten or dime must be transformed to 10 ones or cents and added to the 5 ones or cents; and 1 tenth must be transformed to 10 hundredths and added to the 5 hundredths, before the difference can be found. In a social situation a child who had 7 dimes and 5 cents could not give another child 7 cents until 1 dime was changed (transformed) to 10 cents.

In finding the degree of difference between two numbers, the transformation is as follows:

$$\begin{array}{r} 034 \\ 7\overline{)238} \\ 21 \\ \hline 28 \\ 28 \\ \hline \end{array}$$

In this example the 2 hundreds in the dividend do not contain the divisor, 7 ones, a hundred times, consequently there can be no frequency in the hundreds place of the quotient, and the zero designates this. The 2 hundreds are then transformed to 20 tens and added to the 3 tens. Here the 20 tens are known and written as 2 hundreds but considered as 20 tens for computation purposes. 23 tens contain 7 ones 3 tens times with 2 tens over. These 2 remaining tens are transformed to 20 ones, and together with the 8 ones equal 28 ones. 28 ones contain 7 ones 4 ones times. The zero in the quotient may later be omitted.

Thus we see that in finding the difference or ratio of difference between two numbers, the transformation is from larger to smaller units, the reverse of that employed in counting, addition, and multiplication.

Let us now consider how the inability to compute dissimilar kinds and sizes of units was overcome. It was difficult to add 4 apples and 5 pears until the abstraction "fruit" was invented. This made it possible to think in terms of one specific abstract unit rather than of two dissimilar tangible units, and when 4 apples and 5 pears are transformed to the common unit fruit, the sum, 9, is easily conceptualized. When a primitive man wanted to trade his flock of sheep he had to barter one at a time and guess at its weight and value, because he could not conceive of a large number of sheep as discrete individuals, and had no units of weight and value. In a modern social situation of this kind the flock of sheep, representing many dissimilar units, are transformed to a common unit of weight and then to a common unit of value in order to make the transaction equitable to both seller and buyer.

Transformation to a common abstract unit is not only required in the computation of quantities of natural units, but also in the computation of dissimilar units of measure. It is impossible to add a pint and a quart until both units are transformed to a common unit, pints; or to add a nickel and a dime until both have been transformed to cents. In adding $\frac{1}{3}$ and $\frac{1}{4}$, both must be transformed to the common unit, twelfths, to get the sum. It is necessary at this point to remember that any abstract unit with which we deal quantitatively, regardless of size or kind, is an integral unit. We know by inference that $\frac{1}{3}$ is larger than $\frac{1}{4}$ because one of the three parts of any quantity is larger than one of the four parts of the same quantity. Since thirds and fourths by inference each represent a definite part, they may also be considered as units. Therefore, in the example $\frac{3}{4} \div \frac{2}{3}$, it becomes understandable to transform to the common unit, twelfths, resulting in

9/12÷8/12. Any frequency of 9 units divided by a frequency of 8 of the same unit equals 9/8 or 1 1/8, because all units are in reality whole units and can be computed as such. This process is meaningful and superior to the mechanical manipulation: $3/4 \div 2/3 = 3/4 \times 3/2 = 9/8 = 1\ 1/8$. Current methods of teaching common fractions by rule manipulation leave much to be desired.[7]

In order to transform two different systems of notation, as found in mixed numbers, to a specific system, for example, to common fractions, the horizontal method of teaching addition and finding the difference is preferable because it can be rationalized, and it conforms to the structure necessary in multiplying and dividing common fractions. In computing the mixed numbers $2\frac{1}{2}$ and $1\frac{1}{5}$, by transforming to the common fraction notation and then to a common unit, we rationalize as follows:

$$2\tfrac{1}{2} + 1\tfrac{1}{5} = \tfrac{5}{2} + \tfrac{6}{5} = \tfrac{25}{10} + \tfrac{12}{10} = \tfrac{37}{10} = 3\tfrac{7}{10}.$$
$$2\tfrac{1}{2} - 1\tfrac{1}{5} = \tfrac{5}{2} - \tfrac{6}{5} = \tfrac{25}{10} - \tfrac{12}{10} = \tfrac{13}{10} = 1\tfrac{3}{10}.$$
$$2\tfrac{1}{2} \times 1\tfrac{1}{5} = \tfrac{5}{2} \times \tfrac{6}{5} = \tfrac{1}{2} \text{ of } \tfrac{1}{5} = \tfrac{1}{10},\ \times 5 = \tfrac{5}{10} \times 6 = \tfrac{30}{10} = 3.$$
$$2\tfrac{1}{2} \div 1\tfrac{1}{5} = \tfrac{5}{2} \div \tfrac{6}{5} = \tfrac{25}{10} \div \tfrac{12}{10} = \tfrac{25}{12} = 2\tfrac{1}{2}.$$

This structure is also a preparation for the horizontal notation required in higher mathematics.

The above mixed numbers could also be transformed to the decimal system of notation and written as 2.5 and 1.2.

It is now clear that transformation is an invention which aids rationalization and thereby helps to overcome the mental handicaps previously referred to. In counting, addition, and multiplication, small units are transformed to larger ones; in finding the difference or ratio of difference between numbers, large units are transformed to smaller ones; and in working with dissimilar units they are transformed to a common unit. Transformation thus changes the form of expression without altering values or ratios.

[7] *Difficulties Encountered by College Freshmen in Fractions.* Walter S. Guiler, Journal of Educ. Research, Oct., 1945, p. 102.

Let us now see how transformation may be applied to the division of decimal numbers. When examples are classified according to the processes involved, they may be arranged and taught in the order of their increasing complexity, so as to take advantage of transfer of training. When, however, their classification is based upon the mere inspection of completed examples, as to whether or not the divisor, dividend, or quotient contain fractions, it is difficult if not impossible, to discover example types, or to arrange them in a scale of complexity. When elements other than those required to understand the placing of the decimal point are considered important for the classification of types, such, for example, as the zero difficulty and errors in the functions previously taught but not mastered, that is, errors in subtraction, multiplication, and division, then the number of supposed types will differ with each investigator, since no constant by which to judge complexity was employed.

There is, however, such a constant by which examples in the division of numbers containing decimal fractions can be arranged in the order of their increasing complexity. It is found in the new processes required to understand and correctly place the decimal point. To recognize these we must first isolate those processes already taught in addition, subtraction, multiplication, and division. This is accomplished by a previous mastery of long division before the teaching of division of numbers containing decimal fractions is begun, and by providing a method for the isolation of chance errors in long division. The latter is done by finally testing children on five examples of each type on the supposition that the pupils successfully completing three out of five examples of a type have satisfactorily mastered the

type, and that the two misses were due to chance error.[8]

The new concepts and processes to be learned are:

a. All decimal numbers are made up of frequencies of whole units that differ only in their relative sizes.
b. The decimal point occupies a fixed position to show where ones end and tenths begin. It enables us to know the relative sizes of the various decimal frequencies which make up a number.
c. Each unit frequency in the dividend must be divided by the whole divisor; the zeros in the quotient, if any, show when that is impossible, as:

$$4/.0002 = 4/.00020 \quad \begin{array}{r} .00005 \\ 20 \end{array}$$

or

$$6/486.24 = 6/486.24 \quad \begin{array}{r} 081.04 \\ 48 \\ \hline 6 \\ 6 \\ \hline 24 \\ 24 \end{array}$$

When the purpose and the meaning of the zero is understood, pupils may discontinue writing it to the left of the decimal point when it precedes the frequency of the largest decimal unit in the quotient, since it does not contribute to the numerical value of a number. In the initial teaching situation it should be employed to represent a process.

As previously stated, examples may be classified either by inspection according to the similarity or difference of their appearance, or on the basis of the new processes required to compute them. When arranged in a scale of apparent difficulty by mere inspection there are nineteen possible structural variations of examples in the division of numbers containing decimal fractions. When, however, they are classified according to the new processes involved, these nineteen apparent variations fall into three process types as shown below. These three types should be taught in the order given, that is, in the order of the increasing complexity of the transformation required, so as to take advantage of the principle of transfer of training.

[8] *Constancy of Errors to Basic Facts in the Fundamental Operation of Arithmetic.* Foster E. Grossnickle, Journal of Educational Research, Jan., 1939, pp. 336–344.

TYPE ONE

Transformation from Larger to Smaller Units.

Quotient a Mixed Decimal

1. Integer ÷ Integer

$$4/30 = 4/30 \quad \begin{array}{r} 07.5 \\ 28 \\ \hline 20 \\ 20 \end{array}$$

2. M. Decimal ÷ Integer

$$4/8.2 = 4/8.2 \quad \begin{array}{r} 2.05 \\ 8 \\ \hline 20 \\ 20 \end{array}$$

Quotient a Decimal Fraction

3. Integer ÷ Integer

$$8/4 = 8/4.0 \quad \begin{array}{r} 0.5 \\ 4\ 0 \end{array}$$

4. D. Fraction ÷ Integer

$$4/.8 = 4/.8 \quad \begin{array}{r} .2 \\ 8 \end{array}$$

5. M. Decimal ÷ Integer

$$4/2.6 = 4/2.6 \quad \begin{array}{r} 0.65 \\ 2\ 4 \\ \hline 20 \\ 20 \end{array}$$

TYPE TWO

Transformation to Frequencies of a Common Decimal Unit

Quotient an Integer

6. Integer ÷ D. Fraction

$$.3\overline{)\frac{3}{\frac{30}{9}}}$$
$$\underline{9}$$
$$0$$

7. D. Fraction ÷ D. Fraction

$$.3\overline{/.9=3/9}$$
$$\underline{9}$$

8. M. Decimal ÷ D. Fraction

$$.8\overline{/2.4=8/24}$$
$$\underline{24}$$

9. Integer ÷ M. Decimal

$$3.5\overline{/7=35/70}$$
$$\underline{70}$$

10. M. Decimal ÷ M. Decimal

$$2.3\overline{/4.6=23/46}$$
$$\underline{46}$$

TYPE THREE

Transformation to Frequencies of a Common Decimal Unit together with Transformation of Frequencies of Larger to Frequencies of Smaller Decimal Units.

Quotient a Mixed Decimal

11. Integer ÷ D. Fraction

$$.4\overline{/7=4/70}$$
$$\begin{array}{r}17.5\\ \hline 4\\ \hline 30\\ 28\\ \hline 20\\ \underline{20}\end{array}$$

12. D. Fraction ÷ D. Fraction

$$.2\overline{/.9=2/9}$$
$$\begin{array}{r}4.5\\ \hline 8\\ \hline 10\\ \underline{10}\end{array}$$

13. M. Decimal ÷ D. Fraction

$$.3\overline{/7.14=30/714}$$
$$\begin{array}{r}023.8\\ \hline 60\\ \hline 114\\ 90\\ \hline 240\\ \underline{240}\end{array}$$

14. Integer ÷ M. Decimal

$$2.5\overline{/7=25/70}$$
$$\begin{array}{r}02.8\\ \hline 50\\ \hline 200\\ \underline{200}\end{array}$$

15. M. Decimal ÷ M. Decimal

$$3.5\overline{/7.7=35/77}$$
$$\begin{array}{r}02.2\\ \hline 70\\ \hline 70\\ \underline{70}\end{array}$$

Quotient a Decimal Fraction

16. D. Fraction ÷ D. Fraction

$$.4\overline{/.02=40/2.00}$$
$$\begin{array}{r}0.05\\ \hline 2\ 00\end{array}$$

17. Integer ÷ M. Decimal

$$2.5\overline{/2=25/20.0}$$
$$\begin{array}{r}00.8\\ \hline 20.0\end{array}$$

18. D. Fraction ÷ M. Decimal

$$4.5\overline{/.9=45/9.0}$$
$$\begin{array}{r}0.2\\ \hline 9\ 0\end{array}$$

19. M. Decimal ÷ M. Decimal

$$6.6\overline{/3.3=66/33.0}$$
$$\begin{array}{r}00.5\\ \hline 33\ 0\end{array}$$

Below are the tabulated results of tests given on the three types of division examples. The class was tested on five examples of each type after the type was taught. A final test was given on all three types, composed of five examples of each type worked from left to right so that each succeeding example was of a differert type. The class received twenty-two forty minute periods of instruction. Due to the pressure of time the children received no previous training in the multiplication of decimal numbers. This will now be taken care of that this study is completed. The results obtained cannot be credited alone to the instruction given during the twenty-two periods because all instruction in this and previous grades emphasized meanings. The class was able to do long division before this experiment began.

Final Test Results

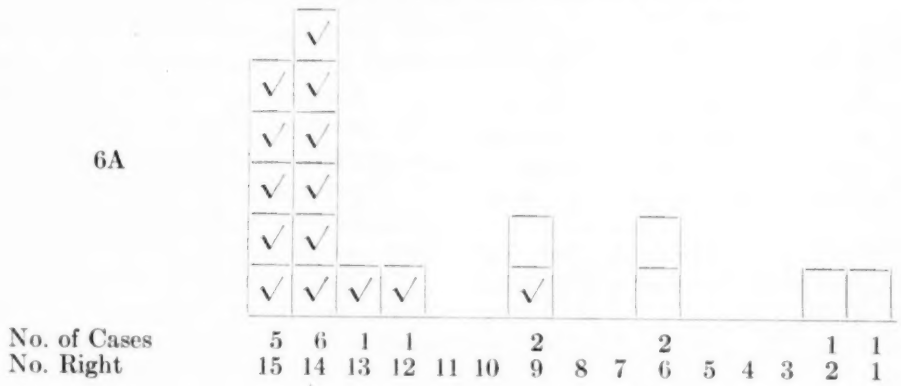
6A Pupils	I.Q.	Tests on Indiv. Types—No. Right			Tests on Combined Types—No. Right			
		1	2	3	1	2	3	Total:
1	131	5	5	4	4	5	5	14
2	128	5	5	4	5	5	5	15
3	127	5	4	5	5	5	5	15
4	121	5	A	A	A	A	A	A
5	121	5	3	4	5	4	4	13
6	120	5	5	5	5	5	4	14
7	119	A	3	0	A	A	A	A
8	118	5	5	5	5	5	4	14
9	114	5	2	2	4	1	1	6
10	111	5	5	5	5	4	5	14
11	108	5	5	2	2	2	2	6
12	107	5	5	5	5	5	5	15
13	106	5	3	1	1	0	0	1
14	105	5	4	4	5	5	5	15
15	101	5	3	0	4	4	4	12
16	100	5	4	4	5	4	4	13
17	97	5	3	3	1	1	0	2
18	96	5	5	5	2	4	3	9
19	91	5	5	4	4	5	5	14
20	91	5	2	3	5	5	5	15
21	91	5	5	A	3	3	3	9

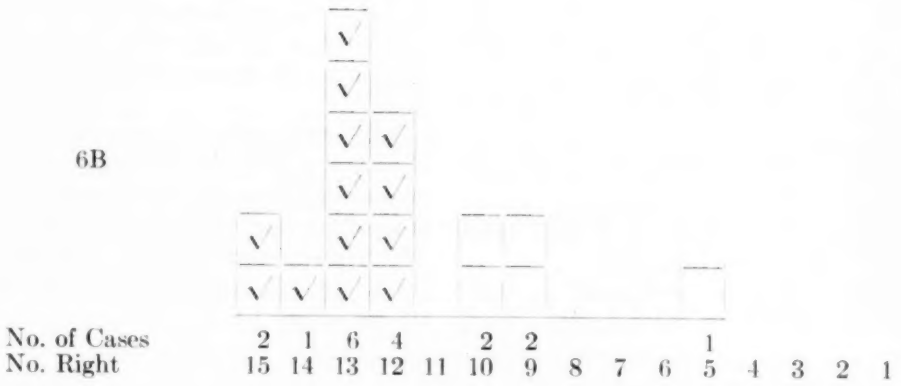
6B Pupils	I.Q.	Tests on Indiv. Types—No. Right			Tests on Combined Types—No. Right			
		1	2	3	1	2	3	Total
1	131	5	4	5	5	4	4	13
2	124	A	4	0	4	5	3	12
3	111	A	5	3	3	5	5	13
4	110	5	4	0	2	2	1	5
5	110	5	5	5	4	5	5	14
6	109	5	3	4	5	2	3	10
7	109	5	5	5	3	5	5	13
8	109	5	5	4	5	5	5	15
9	108	5	5	5	3	5	5	13
10	105	4	2	1	A	A	A	A
11	104	5	3	3	4	3	2	9
12	104	5	4	4	3	5	5	13
13	103	5	2	2	5	2	2	9
14	102	5	5	5	5	5	5	15
15	100	5	2	4	4	4	4	12
16	97	5	4	1	3	5	5	13
17	97	5	5	4	3	4	5	12
18	90	4	5	2	3	4	5	12
19	81	5	4	0	2	4	4	10

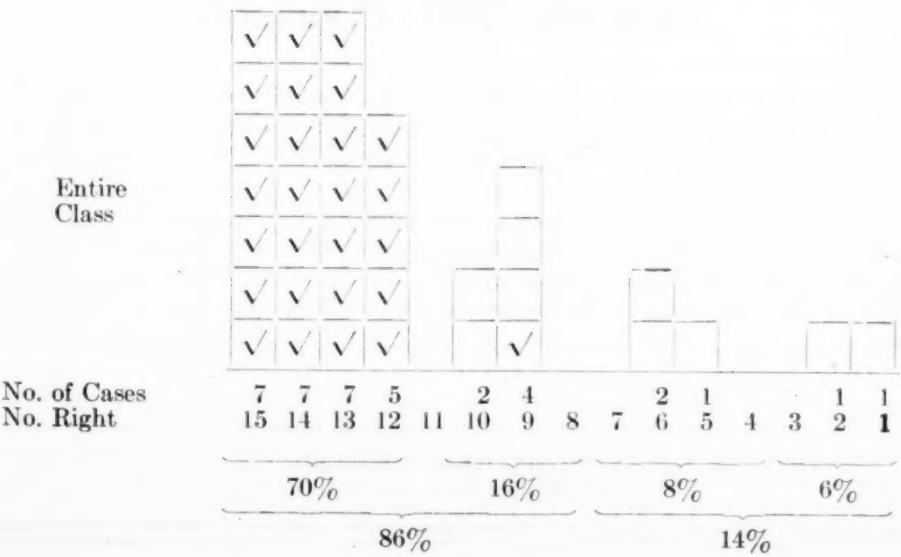
The examples were kept simple because there is no purpose in subjecting children to complex and arduous computations.[9] It is best to avoid possible errors due to fatigue and unnecessary complexity. This enables both teacher and class to concentrate on the meaning and placement of the decimal point. It should also be noted from an examination of the following tabulations that when meanings are emphasized the I.Q. loses some of its significance, and that, as in reading, some children with good intelligence quotients have difficulty with thinking in arithmetic. Some of these would perhaps have done better temporarily had they been permitted to memorize and manipulate rules. When, however, critical thinking is required, those pupils with a good memory only are not given undue advantage, and

[9] *Some Factors Affecting A Test Score in Division of Decimals.* Foster E. Grossnickle, Journal of Educational Research, Jan., 1944, pp. 338–42.

Graphs of the Final Test
On the Three Types of Decimal Division Examples







those who are slow often prove to be good thinkers.

Final Test

Type One	*Type Two*	*Type Three*
$5\overline{)54}$	$.04\overline{)12}$	$.6\overline{)8}$
$8\overline{)8.3}$	$.002\overline{).8}$	$.04\overline{).9}$
$15\overline{)3}$	$.03\overline{)1.5}$	$.07\overline{)8.51}$
$9\overline{).180}$	$2.4\overline{)9.6}$	$7.3\overline{)8}$
$.7\overline{)5.134}$	$2.6\overline{)78.52}$	$8.9\overline{)9.2}$

Children were tested on each type taught and finally tested on all three types, worked from left to right so that each succeeding example was of a different type. Five examples of each type were included. It is assumed that the child mastered the type when he got three out of five of each type right.

The checked squares in the above graphs represent pupils who got 3 out of 5 examples of each type correct.

27 pupils, or 73% had 3 or more out of 5 of each type correct.

32 pupils, or 86% had 9 or more out of 15 examples correct.

5 children who got 9 out of 15 right failed to get 3 out of each type right.

5 children may be considered to have definitely failed to master division of decimal numbers up to the time of the final test. However, the unfavorable results obtained by these 5 children were not due to low intelligence but rather, it seems, to other undesirable traits, such as poor retention and lack of concentration and attention. They seemed to understand what was required of them during the instruction period, but were unable to make good on the final test. They may be more successful by the end of the semester.

The writer is satisfied that with a little more instruction and practice these children will have been well taught in the division of decimal numbers. If at the end of high school or during their freshman year at college they are retested and the results are poor, such results will probably be due to the fact that arithmetic was discontinued at the high school level or that meanings were displaced by rule manipulation.

When the arithmetic curriculum in the elementary school is lightened, time will be provided for instruction in meanings, relationships, and critical thinking. This will produce bettter results than are currently secured, especially if continued on the secondary level.

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The One World of Mathematics

By JOHN KINSELLA
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and
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Hunter College of the City of New York

ONE doctrine proposed by the agressors in World War II was that of racial superiority. These agressors became the losers. A proposition of the victors who earnestly desire a permanent world peace is that the nations of our planet must adopt cooperation rather than "survival of the fittest" to guide their relationships. Only in this way, they say, can we become winners of the peace. On the other hand, the bickering and quarreling among the great powers about atomic bombs, boundaries, and the other terms of peace for the Hitlerian group seem in direct violation of the proposed peace recipe.

The interdependence of these nations economically, spiritually and intellectually is commonly accepted. That the invention of the wireless, the airplane and other means of communication has brought men in all sections of the world closer together is already a trite idea. Yet the dealings of our leaders might lead a visitor from another world to observe that we were acting on an assumption of an earth made of island nations.

The early history of mathematics frequently reveals a kind of isolation, too. The lack of efficient means of world intercommunication provided an alibi for much of this. Jealousy and secrecy, though, was responsible for a large part of the blame. The creator of mathematical ideas feared that the transmission of them to others would rob him of the honor of invention or discovery. Even if he had a glimpse of the concept of service to mankind, he felt more strongly that his personal satisfaction and vanity would receive most nourishment from a small circle of competing workers and admirers.

On the other hand, the history of mathematics viewed in broad outline portrays the inter-dependence and relationship of cultures. Professor Archibald in his *Outline of the History of Mathematics* considers mathematical history previous to 1600 in four periods: Babylonian and Egyptian, 3500 B.C. to the beginning of the Christian era; Grecian, 600 B.C. to 500 A.D. Hindu, Arabic and Persian, 500 to 1200; European 1200 to 1600.[1] The mere mention of these divisions indicates that mathematical developments have not been the monopoly of one race or nation. Granting that students are not impressed by the minutiae of mathematical history, dates, names, titles, and moot questions of priority, is there not something of cultural value and intrinsic interest in this panorama of development?

History does not record the names of individual Babylonian and Egyptian mathematicians, nor does it record in detail the cultural interchanges between the great civilizations of the Near East and the Orient. Recent discoveries, notably those of Dr. Otto Neugebauer, have shown an astonishing development of mathematics among the ancient Babylonians. While they did not reach literal algebra, their solutions of numerous geometrical problems, involving quadratic equations were essentially algebraic. Babylonian mathematics is characterized by numerous tables; reciprocals, products, squares, cubes, and higher powers. Professor Karpinski has shown that Euclid's *Data* repeats a number of Babylonian

[1] Archibald, R. C., *Outline of the History of Mathematics*, Mathematical Association of America, 5th edition, 1941.

problems in a manner which cannot be explained by mere coincidence.[2]

One matter of fundamental interest is the distinction between the Babylonian and Egyptian method of writing numbers. The Babylonians used a place value system with base 60. Our decimal place value notation requires ten symbols and our numbers are of course polynomials of the form,

$$\cdots a_2 10^2 + a_1 10 + a_0 + b_1 10^{-1} + b_2 10^{-2} + \cdots$$

where the coefficients are numbers from 0 to 9. The Babylonian numbers were of the form,

$$\cdots + a_2 60^2 + a_1 60 + a_0 + b_1 60^{-1} + b_2 60^{-2} + \cdots$$

where the coefficients were numbers from 0 to 59. The Babylonian system was not perfect since they lacked a symbol for zero and a means of indicating the "decimal point." However even with its imperfections, the Babylonian number system was superior to the other number systems of antiquity and the Babylonian base of sixty is preserved in our measures of angles and time. The Egyptian number system was decimal but not place value, that is, successive powers of ten were represented not by the same symbol in different positions, but by different symbols. The Egyptian represented the number 987 by repeating the units symbol seven times, the tens symbol eight times and the symbol for one hundred nine times. The advantages of the place value system are obvious, but our present system, developed by the Hindus and transmitted by the Arabs was adopted by the Europeans with reluctance.

The cosmopolitan character of mathematical development is strikingly shown in the history of Greek mathematics. David Eugene Smith lists forty-five localities important in the history of Greek and Roman mathematics, including five in Egypt, six in Palestine and Syria, and nine in Asia Minor. Dr. Smith says, "Indeed contrary to the idea that is commonly expressed, Greece always depended largely upon external influences for her mathematics, and few who advanced this science in her schools were born within her continental areas."[3] Dr. Sarton in his article on the *Unity and Diversity of the Mediterranean World* gives an impressive list of thirty-four Orientals prominent in Grecian science between the second century B.C. and the fourth century A.D. In Dr. Sarton's list are the following well known mathematicians and astronomers: Hero, Menelaus, Ptolemy, Pappos, and Theon from Alexandria; Nicomachos of Gerasa; Hipparchus of Nicaea; Theon of Smyrna; and Geminos from Rhodes. In conclusion Dr. Sarton notes "Thus was a curious cycle completed, for the beginning of Greek science had been Greco-oriental as well as its end."[4]

Certain periods in scientific history are pre-eminently times of transmission of knowledge from one group to another. Such a period is the beginning of Muslim science under the Abbāsid caliphs of Bagdad, notably Al-Mansūr (754–775), Hārūn al-Rashìd (786–809) and Al-Mamūn (813–833). These caliphs secured the services of competent scholars in order to translate the then available science into Arabic and so laid the basis for scientific work in the lands under Muslim rule. To quote Dr. Sarton again "Every bit of scientific knowledge had to be translated from Pahlavi, Greek or Sanskrit. Scientific and philosophic terminology had to be created. The Abbāsid court was permeated with Persian, Jewish, and Nestorian influence with Persian influence predominating. . . . All understood Arabic, but also spoke or read Persian, Syriac, Sanskrit, Hebrew or Greek."[5] The measurements

[2] Karpinski, L. C., *Origin of Freshman Mathematics*, Scripta Mathematica, Vol. VI (1939), 133–140.

[3] Smith, D. E., *History of Mathematics*, Vol. 1, 55, 59–61.

[4] Sarton, George, *The Unity and Diversity of the Mediterranean World*, Osiris, Vol. II (1936), 429–430.

[5] Sarton, George, *Outline of the History of Science*, Vol. I, 528 ff.

for laying out Bagdad were made by Al-Naubakht, a Persian astronomer and engineer, and Māshallāh, an Egyptian Jew. Māshallāh's work on the astrolable formed the basis of two manuscripts by Rabbi ben Ezra (c. 1140) and from one of these Chaucer seems to have drawn his information for his treatise on the astrolabe.[6] The cosmopolitan character of Muslim science did not end with the initial period of translation and assimilation. The Bakntyashū family, Nestorians from Persia, were physicians to a long line of caliphs and exerted a deep influence on Muslim medicine in the eighth and ninth centuries. Omar Khayyam, known in the West as a Persian poet and philosopher, was primarily a mathematician and astronomer, and despite his unorthodox religious views, Arab historians and astronomers pay unstinted tribute to his scientific work.[7] Omar made a systematic study of cubic equations and developed a calendar which was more accurate than the Gregorian.

One is not accustomed to think of the Mongols as patrons of science, yet the following statement of H. G. Wells is not without definite substantiation. "We hear too much in history of the campaigns and massacres of the Mongols and not enough of their indubitable curiosity and zest for learning. Not perhaps as an originative people, but as transmitters of knowledge and method their influence upon the world's history has been enormous."[8] In 1259 the Mongol conqueror and ruler of Persia, Syria and Asia Minor gave Nāsir al-din the means to construct an observatory and library at Marāgha in Adharbāyjān. For a short time this was an outstanding center with astronomers, mathematicians, and instrument makers from the surrounding countries, the Caucasus, Morocco, Christian Syria and China.[9] Kubalai Khān drew scientific assistants from many countries. Īsa the Mongol (Ai-hsieh in China), a Nestorian, was head of Kubalai's astronomical board. A Persian Jamāl āl dīn (Cha-ma-li-ting) devised a new calendar for Kubalai and introduced Persian astronomical instruments in the East. Arab engineers were in the Mongol service in 1270 and Muslim spherical trigonometry was introduced into China by Kuo-Shou-Ching.

A second great period of translation began in the twelfth century when Greek, Hindu and Arabic science was made known to Europe through translations from Arabic to Latin. The first Latin translations of Euclid, those of Adelard of Bath and Gherard of Cremona were made from the Arabic not the Greek. The task of translation is not easy and seldom receives its proper recognition; frequently it was necessary for two translators to work together and make the translation from Arabic to Latin through the medium of a third language. Thus John of Spain and Domingo Gundisalvo worked together, John translating from Arabic to Castilian and Domingo from Castilian to Latin. Plato of Tivoli and Abraham bar Hiyya (Savasorda) used Hebrew as a medium from Arabic to Latin. The importance of Jewish scholars at this time is of interest, Dr. Smith says, " . . . the Jews contributed in no small degree to the advance of mathematics in Spain, and to them the Christians were indebted for the first knowledge of the Arabic works on the subject."[10] Sicily occupied a unique position in this period of translation and transfer; here Greek, Latin and Muslim cultures dwelt together in comparative amity. Under the Norman rulers of Sicily the court language was French and business was conducted in Greek, Arabic, Latin and Hebrew. Ptolemy's *Almagest* was translated in Sicily some years before the more well known translation by Gherard of Cremona.

The scene of mathematical activity in

[6] Smith, D. E., *History*, Vol. 1, 169–170.

[7] Kasir, Daoud S., *The Algebra of Omar Khayyam*, New York, 1931, 3, 4.

[8] Wells, H. G., *Outline of History*, 675.

[9] Sarton, George, *Outline*, Vol. II, 754.

[10] Smith, D. E., *History*, Vol. I, 206.

. . Europe shifts from place to place with distinguished contributions from practically all countries. More than two hundred mathematical works were published in Italy before 1500, but this number was increased to 1527 in the next century.[11] In thè sixteenth century algebra was advanced by the Italians Scipio del Ferro, Antonio Fior, Tartaglia, and Girolamo Cardano; trigonometry by the Germans Johann Müller and Georg Rhaeticus; both algebra and trigonometry were immeasurably aided by the literal notation of François Viète, whose work also paved the way for the subsequent development of analytic geometry and the calculus. In *Men of Mathematics*, E. T. Bell discusses the lives and works of those whom he considers the creators of modern advanced mathematics. France is represented by Descartes, Fermat, Pascal, Lagrange, Laplace, Monge, Fourier, Poncelet, Cauchy, Galois, Hermite, and Poincaré; Germany by Leibnitz, Gauss, Jacobi, Weierstrass, Kronecker, Riemann, Kummer, Dedekind, and Cantor; England by Newton, Sylvester, Cayley, and Boole; Switzerland by the Bernoullis and Euler; Russia by Lobatchewsky; Norway by Abel; Ireland by Hamilton.

[11] Archibald, R. C., *Outline*, 26.

Mathematical history has of course been marred by racial and religious persecutions both in recent times and in the past, but needless to say modern mathematics remains international. As Professor Karpinski has said in discussing the origin of freshman mathematics "By this route the Aryans and non-Aryans created the universe of mathematics which we have today, and the freshmen in our universities are the heirs of all the mathematical ages past."[2] By emphasizing the fact that mathematics has been the creation of men of all nations, building on the ideas of their contemporaries as well as their predecessors, teachers of mathematics have an opportunity to slow, to some extent, the growth of the idea of super-races and to influence in a modest way a type of national action guided by a concept of world interdependence.

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EDITORIALS

Guidance Report of the Commission on Post War Plans

THE leading article in this issue of THE MATHEMATICS TEACHER is the final report of the Commission on Post-War Plans of the National Council of Teachers of Mathematics. It is the result of a great deal of hard work on the part of the members of the Commission and we should all be thankful to them for getting out such an excellent report.

This report is intended for use with high school pupils both by teachers of mathematics and other teachers as well, who may be asked for guidance.

THE MATHEMATICS TEACHER is anxious to have this report distributed as widely as possible. To this end it is making a large number of reprints with covers (called "A Guidance Pamphlet in Mathematics") which will be sent out in quantity to groups who are interested in them for actual cost plus transportation charges.

All persons who are interested in helping youth to discover the place of mathematics in the life work which they select should be interested in this helpful Report.—W. D. R.

The Mathematics Teacher and the Yearbooks

IT IS most unfortunate at this critical time in the affairs of the world that so many former members of the National Council of Teachers of Mathematics (automatic recipients of the official journal) should be so careless or thoughtless about renewing their membership promptly when it expires. In order to carry on successfully in the future we shall need the support of as many individual teachers as possible. We hope that in the future we may receive replies to our renewal notices promptly so as to save greater expense in time and money. Members can help greatly by being prompt with renewals.

The National Council Yearbooks are rapidly going out of print. The supply of the first, second, tenth, eleventh, twelfth, thirteenth, and seventeenth is now exhausted. The best way to make sure that your files are complete is to order each Yearbook when it first appears.—W. D. R.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR

Midwood High School, Brooklyn 10, New York

The American Mathematical Monthly

May 1947, Vol. 54, No. 5

1. Dubisch, Roy, "The Wedderhurn Structure Theorems," pp. 253–259.
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The Australian Mathematics Teacher

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3. Sarjeant, H. B., "Golf from a Mathematical Angle," pp. 3–9.
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7. Reviews, pp. 28–29.
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9. Gleanings (34–39).

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5. Fawcett, Harold, "Unifying Concepts in Mathematics," pp. 59–62.

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Mathematics Magazine (formerly *National Mathematics Magazine*)

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11. Problems and Questions, pp. 58–59.
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13. Our Contributors, pp. 61–62.

The Annual Meeting of the National Council of Teachers of Mathematics will be held in Indianapolis, Indiana, on April 2 and 3, 1947. Headquarters will be at the Hotel Claypool. Better secure your hotel reservations now.